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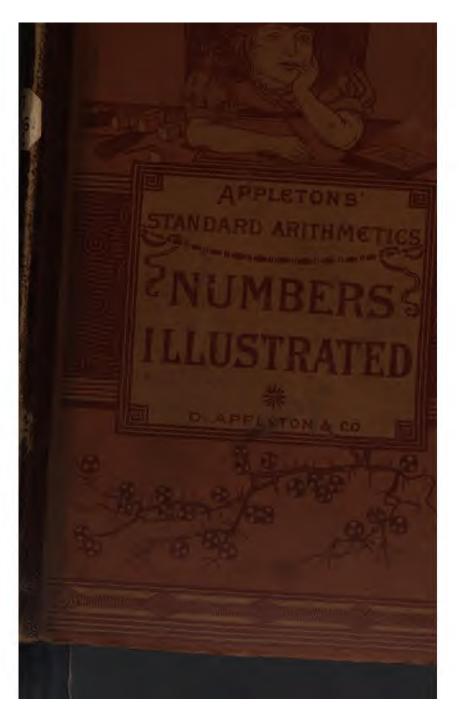
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APPLETONS' STANDARD ARITHMETICS

'NUMBERS ILLUSTRATED

AND APPLIED IN

LANGUAGE, DRAWING, AND READING LESSONS

AN ARITHMETIC FOR PRIMARY SCHOOLS

Andrew J. Rickoff

And

E. C. Davis

NEW YORK, BOSTON, AND CHICAGO

D. APPLETON AND COMPANY
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PREFACE.

It is the design of this book, in the first place, to familiarize the child with numbers and their combinations, not by means of repeating such formulæ as 4 and 3 are 7, but by provoking observation to lead him to the adoption of the formula as a statement of his own experience. In this way an intimate and spontaneous association of thought and expression will be induced, and that lightlessness avoided which makes it possible for many children to repeat 4 and 3 are 7, without a thought of 4 or 3, or of the combination of 4 and 3.

It is not difficult to understand how it is that so much effort is wasted in teaching a child to answer always with readiness and confidence the question, "How many are 4 and 3?" when it is recollected that the custom has too often been, not to lead him to tell us what he has observed and knows, but to repeat a form of words which he had committed to memory as mere words. The methods of our schools are happily greatly improved in this respect, and it is the purpose of this book, especially of Part I, to afford a great variety of exercises in which the pupil may gain a wide experience in the application of number to objects, and a ready perception of their relation to each other.

The pictorial illustrations at the head of the pages, entitled "The Conversation," are designed for language-lessons in which the immediate design is to excite thought and cultivate expression, but their adaptation to the ultimate purpose of the book will be readily understood.

In the smaller illustrations, under the heading "What can you tell?" the imagination is called into more active play, and the child is led to give more independent and original expression to the ideas gained from the pictures than the purpose of the foregoing exercises would permit, which was to follow out a logical and consistent plan of development. Here he is to find for himself the thread of the "story" hinted at in the picture, and weave it into a connected form for himself, the basis being the special combinations suggested in the picture.

In these illustrations, the operations in numbers are more definitely brought to notice than previously, but it is especially desirable that no form of words or even process of objective illustration be repeated with such a degree of uniformity that the mere form may assume undue importance, or supersede the exercise of intelligence.

The Slates are to supplement and carry on the object-work, suggestions for which will be found in the general notes. They serve the twofold purpose: first, of teaching the child the use of the slate, at the same time familiarizing him with the language conveying the ideas, position, direction, etc.; and, second, of picturing out the various combinations with more distinctness and freedom from distracting surroundings than can be attained in any other mode of illustration.

The Diagrams, which, after Lesson V, take the place of the slates, are designed for more extended slate exercises, still involving the use of numbers. Their usefulness as primary drawing-lessons can not fail to be recognized.

The children are not only to discover and describe the various combinations depicted, but, taking them as models, are to exercise their ingenuity in making as many others as possible with marks, dots, etc., upon their slates, or with sticks and other counters. This is a training in form, number, and arrangement, and, if counters of different colors can be obtained, admirable lessons in color may also be given. Thus is the inventive faculty called into play, imagination exercised, and taste cultivated, while the child is becoming accustomed to the number in all its combinations. The constant handling of the number, in this and the other exercises, affords the child experience, and, as far as the purely arithmetical aim of these first lessons is concerned, this is the sole object, not to teach him to say that 2+1+2=5, but to lead him to know it by experience.

Figures and arithmetical signs have, of course, no place in Part I. Normal or schematic representations and names only are given. These are to be learned by sight, as suggested in the general notes.

In the Script Lesson at the foot of each page will be found the name of each number in script form. This is given for copy-work.

Hints.—The suggestions and questions to be found under this heading on the teacher's page are merely intended as hints of an extended work and great variety of exercises, to be supplied at the discretion of the teacher.

The Parts from I to V indicate divisions of the subject; they do not correspond with successive grades in school classification.

PART I.

Suggestions for exercises to precede and supplement the lessons on the illustrated pages.

SLATE EXERCISES.

Object Lesson on S/ate.—As the slate and pencil are most important implements of study in this branch, it is well to make them first objects of study—for one reason, that the child may become so well acquainted with them and their use that his ignorance of the tools may not interfere with the efficiency of the work to be done with them, and, also, because the study of them affords excellent opportunity for certain preparatory work necessary to the introduction of written arithmetic. Although every teacher has, doubtless, her form for object lessons on the slate, yet suggestions are here given for such lessons, that certain points having a more especial bearing upon the subject under consideration may not be overlooked.

The children are first encouraged to tell, in their own way, all they can see or be led to observe about their slates; then, by skillful questioning, not hinting too much as to answers expected, the teacher draws out the following, in consecutive form. Frame.—Use of frame; what made of; if covered, with what, and why [a little lesson on quietness]; if rubber corners, why. For slate part.—Of what made; hard; breakable; color. Parts.—Sides, how many; corners, how many; faces, one looking up at you, upper face; one looking at desk, lower face; how many faces has a slate? How many faces have you? Care of slate.—Breaking, scratching, cleanliness, etc.

An Object Lesson on the Pencil has its place here, and the following points to be made are suggested: A conversation on pencils in general; use lead-pencils for white paper, black on white, stone-pencils for slate, white on black; compare crayon and blackboard. Shape.—Long, round, pretty; easy to hold. Ends.—How many; one blunt, one sharp; why sharp. How to hold pencils.—Position of fingers; of wrist [writing position]. Use.—Make firm, light lines; heavy lines scratch and are not easily erased. Care of pencils.—Easily broken; carefully handled; kept sharp for neat work and light lines.

Lessons on Position and Direction.—It is an accepted theory that the child must go from the known to the unknown, and that each newly-acquired experience be made a stepping-stone to the next. Few children on first entering school, without previous training, would be able to obey the direction, "Make a ring in the left upper corner of your slate." On the other hand, few, if any, would be found that did not know which is the right hand. With this bit of "terra firma" to stand upon, the teacher, after giving a short and entertaining exercise upon "right hand and left hand," begins the lessons on the use of the slate by the following exercise, which has for its purpose the learning of

Position.—The children being first directed to place their slates lengthwise on their desks [the word will be readily learned by "telling and doing"], the teacher begins by having them point out the right side of the slate; left side; the lower side [side lower down on desk]; upper side [side farther up on the desk]. "How many corners on left side? Where are they? Who can point to a left corner? To the left upper corner? To the left lower corner? Who can tell me the name of this corner [pointing to corner of slate pictured on blackboard]? Who can find the right upper corner? Who can find the right lower corner? Jennie may point to any one corner of her slate and tell me its name. Johnnie may show and tell another, etc., etc." After this follows an exercise on

Direction.—"Put your finger on the left side of your slate, in the upper corner. Now slowly move it downward as far as you can. Where does it stop [left lower corner]? Where did you start? Now start at the left lower corner, and move your finger upward. Where does it stop?" Repeat with the right side. "Put your finger in the left upper corner again. Now move it slowly across the top of your slate, toward the right side. Where does it stop [right upper corner]? Where did you start? Now start at the right upper corner, and move it back toward the left." Question as before, and then repeat with the lower side of the slate.

CLASS EXERCISES.

THE Class Exercises outlined here are given only suggestively, to be amplified or condensed, omitted or repeated, at the discretion of the teacher; their purpose being, mainly, to show what kind of oral work may advantageously supplement the work for the children's pages.

By means of such exercises as the following, each of which has its definite purpose, it will be found that memory is cultivated, imagination stimulated, and observation trained; sight, hearing, and touch are exercised, and thus a harmonious development of the child's powers is attained; the habit of attention, more or less sustained, is formed, and prompt action in obedience to command is acquired, as also a ready expression of thought.

- 1. The teacher calls upon Johnnie to come and find out what she has in her right hand. Johnnie finds "a marble." Jennie finds "two marbles" in her left hand. "Would you rather have Jennie's marbles or Johnnie's? Why? Who would rather have mine [showing a handful]? Why?"
- 2. "How many more marbles has Jennie than Johnnie? What can I do so that Johnnie and Jennie will each have the same number of marbles?
- 3. "Ned, bring me one little girl; now bring me one little girl again. How many times did Ned bring me one girl? [Putting arms around them and bringing them close together.] How many are there? [Sends one to her seat.] How many did I send back? How many are left here?
- 4. "Hold up as many hands as I do. Hold up twice as many. Who can show me this many [two] pencils? Show me half as many."
- 5. "Now, let me see all the little heads bowed down upon the desks. Shut your eyes tight. Listen! [Claps twice.] Wake up! Who can tell me what he heard? How many claps?
- 6. "Who would like to play blindman? Well, blindman, feel these pebbles and tell me how many there are." She tests him with numbers, from one to three, and then with a large number, calling out the expression "many pebbles."
- 7. "Clap your hands this many times. [Makes two rings.] Clap once for each star I make. [Makes ten stars, and covers them quickly.] Tell me, without seeing, how many stars I made. How many times you clapped."
 - 8. "Show me as many counters as you have heads; arms; chins; cheeks."
- 9. "Who knows of something that has one wheel? Two wheels? Three wheels? Two feet? Four feet? More than four feet?"
- 10. "How many eyes has a cat? What has one eye? [Needle.] How many ends has a pin? Name them. How many wings has a bird? A fiy?
- 11. If Nellie earns one penny making lamp-lighters to-day and one penny to-morrow, how many will she have? If you had two pencils, and lost one yesterday and one to-day, how many would you have left?"
- 12. "Nellie, find two blue stars [paper]. Jennie, find one red, one blue, and one yellow star. Walter, find three different colored stars."
 - 13. Who can touch two different things? Three different things?
- 14. "I hear the clock ticking, a bell ringing, and Will writing on his slate. How many sounds do I hear? Who can tell of two different sounds?"

I. Outline of Lesson on the Words and the Schemas.—"What is this, children [pointing to it on the blackboard]?" "A dot." "How many dots?" "One dot." "Say again the word which tells how many dots; say it slowly. It sounds like the tone of a great bell. Would you like to see how it looks?" The teacher prints slowly, and in large letters, the word one, and then calls upon individuals to point to it and pronounce it. Then, "Show me one book; one slate," etc. "Show me [pointing to the word but not pronouncing it] [one] pencil; [one] desk," etc.

In the same way, by showing ••, the word "two" is introduced. "Sounds like the cry of an owl." The teacher then prints the word, as before, and lets the printed form "tell" how many fingers, hands, books, etc., to be shown or pointed to. Changing the questioning from one thing to two things, she tests their conception of the meaning of the two words. Then, showing one and two of various articles, she calls upon first one and then another to point to the word which tells how many things she is showing. The class is then permitted to find these words (and show how many) in chart or readers, or any text at hand, and finally to discover both the words and the schemas in their arithmetics.

II. Hints for Language Lesson on "The Conversation."—The general aim of this exercise is to give a liberal training in reading, and expressing the thought contained in pictures, while the special points to be brought out through this medium are a recognition of the number illustrated, first as a whole and then as to its groupings.

This is to be attained by means of a familiar conversation in which the children are encouraged to give full and free expression to the ideas they gain from the picture, their attention being directed to special points by the questions of the teacher. Since the principal aim of this lesson is to stimulate thought and cultivate the imagination, it would not be advisable to throw the child off the track of the thought by insisting upon accurate description, nor by too frequent corrections of language.

The teacher would do well to first bring the children into sympathy with the beauty and spirit of the scene in this first illustration, by noting the softened light of early dawn, the fleecy clouds, the rising sun, the long shadows, and the signs of awakening life. "How many of you ever saw the sun rise? Who ever saw the moon? The stars? How many stars? How many suns? What are the children doing? Do blackberries grow on trees? What is the man doing? Is this place in the city or country? What else do you see in the picture?" This last will call out an enumeration

of the objects in the picture, from which, together with the above questions, the teacher may skillfully draw out a more or less connected narrative, which will be "the story the picture tells." The special point, number, may be further developed by questions such as the following: "How many hands has the boy? How many has the girl? How many eyes has the boy? Do you see the girl's eyes? How many feet has the boy? How do you know? Choose the colors you would have for this little girl's dress; her apron, etc. How many hats has the girl? The boy? How many hats in the picture? Count the sheep. Count the birds. How many more doors than windows has the barn?" etc.

///. S/ate-work.—In all Recitation Exercises upon the slate, let the conscious effort and ambition of the children be "to tell a nice long story," i. e., to give a full and exact description. The degree of accuracy with which the child describes the slate, and the location and arrangement of the objects thereon, will be a test of the exactness of his observation. With this in view, corrections of language can advisedly be made here, and the exercise may thus serve as a training, both in keenness of perception and accuracy of expression.

In beginning the lesson on the slate in the book, the teacher should first call the attention of the children to the resemblance of this pictured slate to their own slates; also to the differences. "How many corners has your slate? How many has this slate? Are they sharp corners or round corners? How many sides has this slate? Has yours? How many faces? And yours? Point to the left side of this slate; upper side; right side; lower side. Who can tell me what he sees in the middle of this slate?" Require a full and correct statement, first as to "what" and "where," and, after arrangement has been discovered, as to "how" arranged. For example: "I see [or there are] two stars in the upper right corner of the slate. The stars are one under the other." Or "I see two flags, side by side, in the lower left corner of the slate," etc., etc. Every such statement will, of course, have to be at first built up, point by point, because the child is as yet untrained in observation, and does not see it all at once; and, also, because he has not learned how to express what he does see. After all the objects on the slate have been thus "located," comparison as to number is next noted. "How many flags are there? How many more flags than rings? How many stars? Are there more stars than rings?" etc., etc.

A Drawing Exercise may follow, in which the children are directed to either copy on their own slates the objects as they are on the pictured slate,

in their books, or on a similar one which the teacher shall have drawn upon the blackboard.

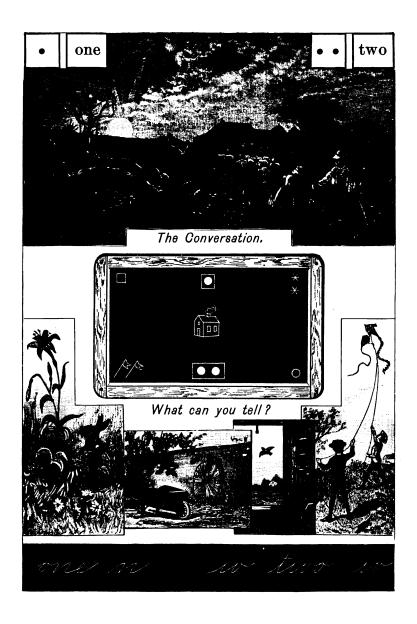
The Dictation Exercises should be introduced by a more or less rapid review of the object-lesson upon the slate. The little workers being ready, with slates in proper position, the teacher directs them to make "a row of stars down the left side of the slate; across the upper side; down right side; across lower side." Then, on the other face, to "make a star in the upper left corner; two rings, side by side, in the lower right corner; a square in the center of the slate," etc., till each place is located and filled.

/V. What can you tell?—In these exercises the children are to be left, as much as possible, first, to give spontaneous and unaided expression to "the story the picture tells"; and, second, to observe the detail with special reference to number. The following are some of the points that may be made in these pictures:

First picture.—A rabbit in field; standing on two hind-legs; holding up two fore-legs; has two long ears; two eyes; we see only one eye, etc. The lily has one open blossom, two buds; half-way up there is only one stem; above, two stems; two leaves on stem with one blossom on it; one leaf on the stem that has two buds, etc. By counting, we find there are ten leaves in all. Second picture.—A garden; wall, with bicycle against it; and a wheelbarrow; vine on wall, etc. Bicycle has two wheels; the wheelbarrow has one wheel (compare); bicycle has two pedals, two handles, one saddle, etc.; wheelbarrow has two handles, two legs, etc. Third picture.—Looking out of the window; a bird-cage; bird flying away; no bird in cage now. "How do you think it happened that the bird got out?" Outside the window a house can be seen, etc. Fourth picture.—Boys flying kites in a field or vacant lot. Two boys, each boy one kite; so two kites—twisted together; each kite one tail; two kites, two tails; same with strings. If one boy runs off with his kite, one boy and one kite left, etc.

V. The Script.—As each school has its established system of writing-lessons, the following brief suggestions are only offered: That the recognition of the written form of the word may be given in the same way as the printed form; that the written and printed forms be compared; that the written word on the blackboard be traced with the pointer, individually, and traced in the air with pencils by the class; and, finally, that it be copied, first as a whole, and then practice given on the accompanying letters.

VI. Hints.—The oral work on One and Two will be found on page 7. For "busy work" hints, see lessons on Three.



- 1. The Word and Schema.—As in Lessons on One and Two.
- //. The Conversation.—After the general conversation the following special points to be made: "How many little girls are there? Names? Each girl has one apple; both want the other one. If (May) takes it, how many will she have? How many more than (Nita)? Which will have twice as many as the other? Which half as many? What can be done so that each will have as much as the other? Look at the picture below and tell."

In the second picture the story will be readily grasped by the children. A few questions only will be needed to give direction to the thought, and bring out the facts in the number.

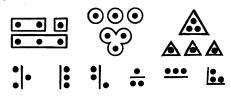
///. S/ate-Work.—Recitation: The pupils to give a full and exact description of each group, as "what," "where," "how many," and "how arranged," as "In the middle of the lower side of this slate there are three flags, two of the flags are side by side, and the other flag is below them." Also compare with slate in Lessons on One and Two.

Drawing: The children may be directed to copy exactly what is on the pictured slate, or, to devise original arrangements of these same objects.

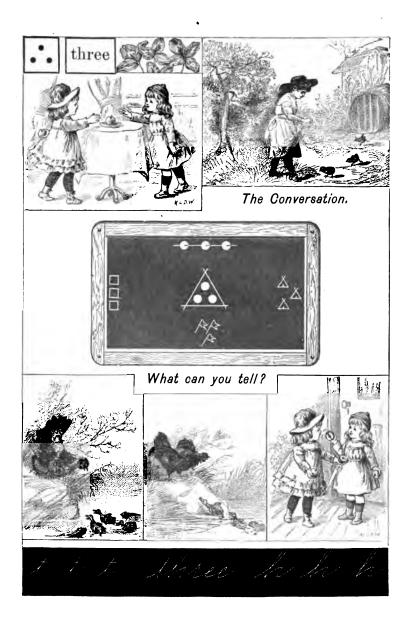
Dictation: Review as in the first lesson. Then dictate from the pictured slate; second, have the groups placed in the corners; and, third, dictate ones and twos of objects to be drawn, as well as threes. In each case have the written work described by individuals.

- /V. What can you tell?—The story of the hen and her three ducklings will be easily gathered by the children from the two pictures. The points in number to be noted are: There are three ducklings and two water-rats. If two rats get one duck each, then two will be taken and one left, etc., etc.
 - V. The Script.—To be taught as directed in Lessons on One and Two.
- VI. Hints for busy work, to be copied from the blackboard by the pupils.

Have the children write or tell number stories about objects in view, as, "There are (2 books) on the (table)



and (1 book) on the (chair). Objects to be represented in outline drawings. "Draw a picture of three things, and write or tell a number story about it; mention three red things seen on the way to school (other colors); three things with wheels; three like things; three different things."

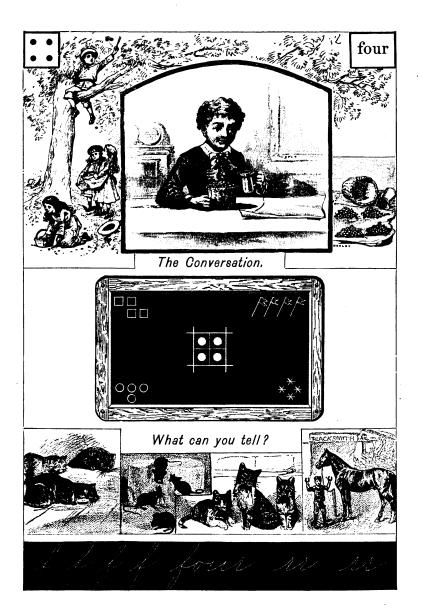


- /. The Word and Schema.—To be given as suggested in preceding lessons.
- //. The Conversation.—After the general conversation upon "The Nutting Party," the following points in number may be noted: 1 child in a tree, 3 on the ground—4 in all. 2 children with hats off, 2 with hats on—4 hats in all. 1 child kneeling, 2 standing, 1 sitting in tree—4 in all, etc. "What do you think the boy in the center picture is going to do? What has he in his hand?" Point to 2 things alike in this picture. Find 3 different things. How many pints of nuts were there in the basket? If he sells 2 pints at 2 cents each, how much money will he get for them?
- ///. S/ate-Work.—Recitation: As in the preceding lessons. Special questions as to number may be, "How many dots in the center? If you erased 1 star, how many would be left? How many times could you erase 2 squares? To how many boys could you give 1 flag each?" etc.

Drawing: Besides the exercises in the preceding lessons, the class may draw picture-slates of the size of that in the book, and exercise their ingenuity in varying the location and arrangement of the pictured objects. A training-lesson in language may be founded upon this exercise, by having each pupil describe in precise terms one of his picture-slates.

Dictation: These lessons would differ from the preceding ones only in the greater variety of exercises possible from the material suggested.

- /V. What can you tell?—The exciting incident pictured in this series of three pictures can not but unloose the tongue of even the shyest child, and "what they can tell" will find spontaneous expression, which need only to be directed. Compare 3 cats, 4 mice. "How many mice get away? Are caught? How many does each cat get? How many fore-feet has a cat? Hind-feet? How many more tails have 4 mice than 3 cats?" etc. In the picture, "The Horse at the Blacksmith's Shop," the points in question are: "How many legs has a horse? A boy? How many more has the horse than the boy? How many times 2 shoes does a horse wear? A boy? Which has twice as many feet as the other? Half as many?" etc.
- V. The Script-Lessons.—To be given as suggested in lesson on One and Two.
- V/. Hints for busy work, see "Three." Have the class of make a picture-story of a boy who had 4 cents, and bought sticks of candy at a cent apiece, as shown here. "If I give Carl one fourth of these four pencils, how many will be get? How many shall I have left? How can I give these 2 apples to four boys? Would you rather have these (4 pennies) or these (2 two-cent pieces)?"

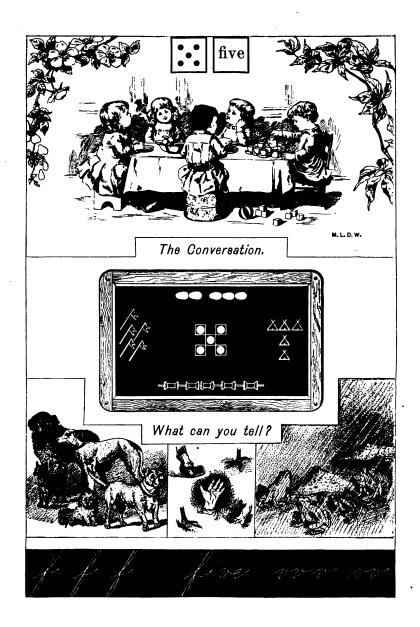


- 1. The Word and Schema.—To be given as suggested in Lessons on Three.
- //. The Conversation.—"Tell me a story of your own about this teaparty" will call out the individual ideas gained from the picture. "How many children are talking? Listening? At the sides of the table? How many cups and saucers? Blocks? If each child takes a block, how many will be taken? How many left? etc., etc.
- "How many leaves on each stem of the ivy? How many 'fives'? Count the ivy-leaves by ones. How many leaves on each stem in the rose-vine? How many blossoms? How many petals in each blossom? I see some rose-petals falling. Who can tell a number-story about this?"
- ///. S/ate-Work.—Recitation: After each group of pictured objects has been fully discussed and all the combinations within five found, compare with the slate for Four to find resemblances; and with that for Three to find differences. Then compare the groups, as the flags with the flags on all the preceding slates, as to number, location, and arrangement.

Drawing: The suggestions given on all the preceding lessons may be put into use here, especially the last one on Four.

Dictation: "Quick-work" exercises in locating and arranging at once and then describing exactly and fully the matter dictated may serve both as a review and as a test of the child's understanding of the terms he has been accumulating in the preceding lessons.

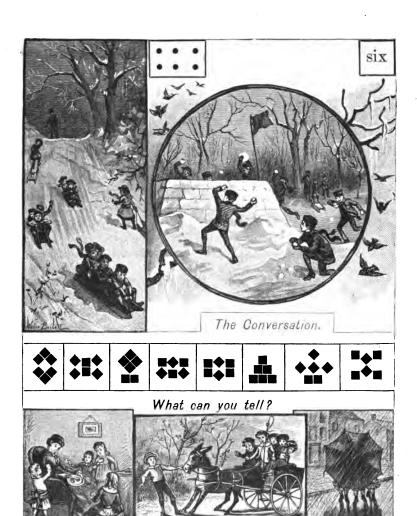
- IV. What can you tell?—"Study the picture, 'The Dog-Show,' and tell me a story about it." Direct the thought by questioning as to the kinds of dogs, the number, etc.: "If the Newfoundland dog should pick up the poodle and carry it off, how many dogs would be left? Suppose a rat should go scampering past, how many dogs would be left? How many tails do you see? How many don't you see? Tell a number-story about each dog." The next illustration was designed for the comparison of "five" with numbers under five, and also for a lesson in comparing the feet of these animals with the human hand. The toads and toadstools illustrate two twos in five, and one left over, etc. "If that little toad goes off, how many will be left? If the wind blows down 1 toadstool, what then?" etc.
- V. The Script.—To be given as heretofore directed, and a comparison made between this word and the preceding ones.
- VI. Hints.—Draw your hand. "If I had 5 cakes, to how many little girls could I give 2 cakes each? John has (showing 5 pennies), and Carl has (showing a nickel), which can buy the most candy?"



- 1. The Word and Schema.—To be given as heretofore suggested.
- //. The Conversation.—"Coasting" and "The Snow Fort" will require but little, if any, questioning to draw out "the story the picture tells" of these the favorite winter sports of childhood. The topic, "Seasons," might also be introduced here.

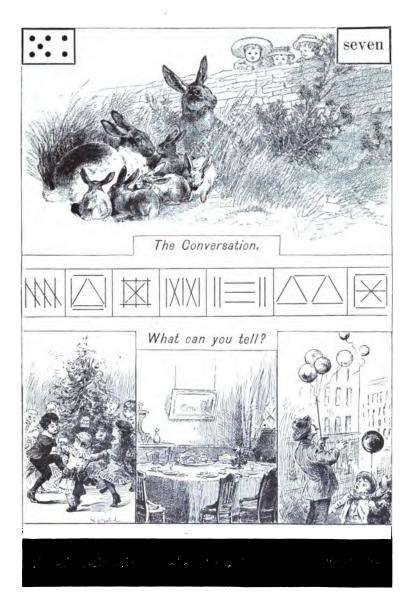
Special points: First picture—"How many sleds? How many boys on the first sled going down? On the second? Altogether? How many girls going down?" etc., etc. Second picture—"How many boys in the fort? In front of the fort?" Other things to be examined as to number: snow-balls, trees, children, birds, and branches of trees outside, etc.

- ///. The Diagrams are, first, to be examined by the pupils to find the combinations in six, and read in class; second, to be reproduced at the desks, either in blocks or in squares of various colored pasteboard, or copied on the slate; third, to serve as models from which the children are to make other and original designs. The inventive faculty will thus be exercised, and training almost without limit, in form, arrangement, and combinations of the number afforded, as also invaluable lessons in color.
- /V. What can you tell?—First picture: "The mother has four children and six cakes. How can she divide the cakes equally, and how much will each child receive?" Other points are, the comparison of other numbers under six. Second picture, the story of "The donkey that wouldn't go"; time of day and season. Special points, "How many children in the cart? Out of the cart? Altogether? On front seat? On back seat? How many feet has the donkey? The boy? Double as many? Half as many? Both together?" etc. Third picture, "The Rainy Day." Make up a little story of your own about this picture. Special points, "How many little girls are there under this umbrella? How do you know? How many pairs of arms? Eyes? Heads?" etc.
 - V. The Script.—Lessons to be given on the plan of the others.
- VI. Hints.—"Draw as many rubber boots as three boys would wear. As many mittens. How many more legs has a spider than an elephant? Than a fish?" etc. A mother said, "I will give you one half of these six cakes if you tell me how many that will be?" "Draw six cents on your slate. Draw as many pencils as you can buy at two cents each." Have the children make six-inch rules—of paper, pasteboard, or wood—for themselves, and encourage them to measure their desks, slates, books, etc.



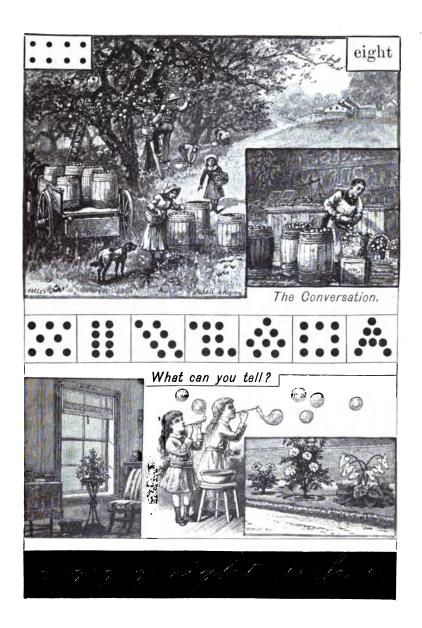
Sist in sin warm

- 1. The Word and Schema.—As previously suggested.
- //. The Conversation.—Upon wild rabbits, pet rabbits, pets in general. Characteristics of rabbits. "Cousin Hare"; fable of "Hare and Tortoise," story of "Brer Rabbit and the Tar-Baby," from "Uncle Remus," adapted by the teacher, are suggested as topics for the general language-lesson. Special points in number upon all the combinations in seven may be made in the groupings of the rabbits.
- ///. The Diagrams may be copied in lines on the slate or represented in objects by match-sticks, splints, etc., and, besides the simple arrangements showing the combinations of number, may represent real objects, such as houses, fences, chairs, tables, flower-pot with plant, and an endless number of like things, and also fancy geometrical designs and figures.
- IV. What can you tell?—The merry circle around the Christmas-tree will arouse the pleasantest recollections, and may be turned to great advantage in inducing the children to relate their own experiences. Heretofore the work in language has been either to describe or narrate. After the general conversation, each child may be called upon to relate an experience which shall have in its plot some combination of seven. The second picture was specially designed to give occasion for a language-object upon "the table," manners, setting a table, etc., while affording excellent material for the study of seven. "Tell all you can about the family who are going to have tea. How many grown people? Children? What ages? Where going to sit?" etc. Special points also on the number of things on the table. The third picture shows an incident familiar and easily described. All the combinations of seven may be noted in the balloons.
 - V. The Script.—As in previous lessons.
- V/. Hints.—"Put seven blocks on your desk. Find another number in these blocks—another." (Six, five, four, three, two two's, and two three's will be found.) Have a pupil tell a number-story, and the class picture it. As "Seven birds were on a fence, and three flew away." Write "Four—and three—are—," and have the children copy, fill blanks, and complete. "Write a number-story of your own about the Christmastree." "How can I divide seven oranges equally between two children? Equally among three? Who can picture it? How many 2-cent stamps can I buy for seven cents?" The children should have breakable objects, and be directed to find one half and one third, also to put together and compare with an undivided whole.

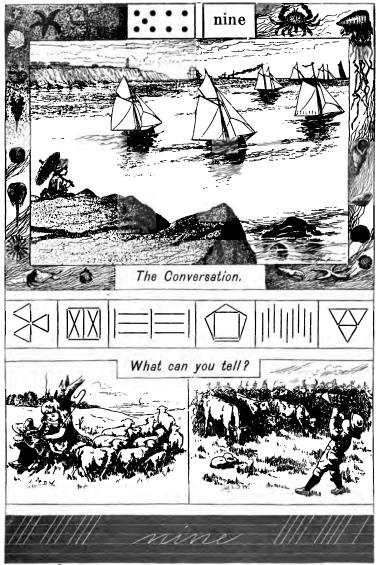


- /. The Word and Schema.—As heretofore, the word as the spoken or written, and the schema as the concrete expression of the number.
- //. The Conversation.—"The Apple-Gathering" affords an effective lesson on country life and occupations. The topic of seasons may be resumed, the teacher taking the children back in memory to the spring-time when these trees were white with fragrant blossoms that spring rains and summer suns have ripened to the round red fruit. Now, in the autumn, the father and his children gather the apples, and early in the morning he goes off to the city and sells them. Here, also, we see the fruit-store of the man who bought the apples. "What is he doing? What measure has he in his hand?" Special points, the grouping of the barrels, the persons, baskets, trees, horses, etc., and the articles in the store.
- ///. The Diagrams are to be used in the same way as those for six. They may be represented in objects by pennies, button-molds, etc., and by variously colored pasteboard disks. The forms may each be read in several different ways, as for instance: No. 4 may be read as either 3 and 2 and 3, or as 1 and 3 and 3 and 1. The class may also be asked to "find other numbers" in the eight. One may find a three, pointing to the upper row in No. 1. The question, "What else is there besides the three?" will call out the fact that 3+?=8. In No. 2, one may find "a four," and immediately discovers that there are "two fours" in the eight spots, etc.
- IV. What can you tell?—The children may "imagine" or "dream" stories about the people to whom this room belongs, two number combinations of eight being required as the basis of each story. Special points, the panes of glass, legs of tables, chair, etc. In the second picture there will be no lack of "stories," which may be given direction by requiring, as before, number to be the basis of them. The third picture is presented for the comparison of the even numbers under eight with eight. "How many more blossoms would you have to add to the first plant to have eight? Which plant has twice as many flowers as another? Three times as many? How many times could you pick two of the lilies?" etc. The comparisons are numerous; only a few types have been given.
 - V. The Script.—To be given as previously suggested.
- VI. Hints.—Name eight different things that can run, hop—that have eyes, ears, hair, fur, wings. Eight kinds of fruit, vegetables, nuts, games.

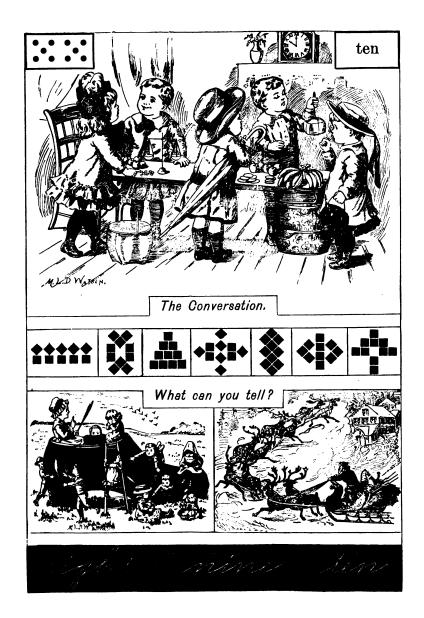
Before the eyes of the children, the teacher folds a paper in four folds, and then cuts out two paper dolls, and asks the children to guess how many.



- 1. The Word and Schema.—To be given as heretofore suggested.
- //. The Conversation.—This beautiful sea-shore scene will afford so many and varied object language-lessons that only a few hints on each of the different lines of thought suggested by it can be given: Land and water—the sea-shore—rocky coast—sandy beach—calm and storm—light-houses—pleasures of the sea—toils of the sea (fishing, whaling, transportation)—ships and boats—the different kinds of motive power—the vessels in the picture—what the sea yields for our use—things under the water—at the bottom of the sea. These last two are illustrated by the frame of the picture, and give opportunity for opening a new world to many of the children. Interesting facts about the wonders and beauties of submarine life may be gathered from almost any natural history. The special points in number may be brought out by questioning on the ship-sails; the legs, etc., of the crab; the points of the star-fish, the sea-anemone, etc., etc.
- ///. The Diagrams.—Too much stress can not be laid upon the value and variety of the exercises which may be founded upon these diagrams. The designs which may be formed, either with long counters or by lines on the slate, are almost without limit, and, if the pupils are required to pay attention to the combinations of number in making their designs, these exercises can not fail to familiarize them with the number in its every aspect-
- /V. What can you tell?—After the story of Little Bo-peep has been rehearsed, require each pupil to tell in what groups he thinks the sheep ran off: one will say, "First three, and then three more, and then three more," and so on, with all the combinations in the number. Again, looking at the illustrations from different points of view, many of the combinations can be seen, as, one and eight, or one and three and five, etc. The story and illustration of "Little Boy Blue" may be treated in a similar way.
- V. The Script.—This lesson introduces a new feature, that of the slanting lines, which serve for practice in slant and distance.
- VI. Hints.—Draw a number-story picture of nine boys and six sleds. Draw another of three rabbits and nine carrots. Draw nine oblique lines (in groups of three, or five and four, etc.). Draw nine horizontal lines; nine vertical lines. Have exercises in making up nine with one, two, three, and five cent pieces. Conduct sales of slate, pencils, sponges, paper-dolls, tops, etc., and have the children picture the operations (as in "Hints," Lessons on Four). Have the children each bring a flower, and find and write number-stories about the parts of the flower.



- I. The Word and Schema.—As heretofore.
- //. The Conversation.—The game "Keeping Store," besides being of unfailing interest to the children, gives special advantages for the study of the combinations within ten, as in buying and selling, exchanging one number of things (pennies) for another, and "making change," etc., both customer and merchant must be ready with his calculations. A study of the articles for sale will also yield most of the combinations. Each pupil may be allowed to make an imaginary sale or purchase of the things pictured here, and tell of the transaction.
- ///. The Diagrams in this lesson are like those in six, and may be used in the same ways. With space less limited far more beautiful designs may be made. It is specially desirable that the pupil should not, in these exercises, be permitted to overlook the element of number, but should note the combinations he forms. In this, and in the other exercises founded upon the diagrams, may be produced what Froebel calls "forms of life—such as actually exist, and come under our observation as works of art and industry; forms of knowledge—such as relate to number, order, proportion, etc.; and forms of beauty—representing ideal forms, models of symmetry and order."
- /V. What can you tell?—If the children have been duly encouraged in their previous work to give a numerical turn to the stories they tell, the teacher will find no difficulty in gathering from the various versions which they will give of "The old woman who had so many children," etc., and "St. Nicholas," the many combinations within ten—combinations which will cover the ground of the four rules of addition, multiplication, subtraction, and division, without, however, having these recognized by the children as distinct operations.
- V. The Script consists of a lesson on "ten," with a review of "eight" and "nine." A review should also be made of all the previous script-lessons.
- VI. Hints.—Make a paper disk and perform operations in fractions before the class, and have them tell what you have done. Then let each child make at home a "paper cake," and then have them divide their "cakes" in halves, fourths, eighths. This will afford many interesting exercises. At another time the cakes may be divided into thirds and sixths. By comparison, and by putting parts together, without formulating what they do, they will be unconsciously learning to perform with parts all the operations that they do with wholes.



- Part //.—*Lessons I to X. The main purpose of these lessons is to introduce the child to the printed forms of arithmetical expressions. They do not embrace all, nor the majority, of the combinations within ten, but may be taken as types for blackboard exercises, in which are introduced the other combinations. They constitute a systematic series of readinglessons, and may besides be copied on the slate, the children filling the blanks with pictures of the objects, and the names of the numbers, thus serving as writing and drawing lessons also.
- *Lessons VI, VII, and VIII.—"Making up and writing original problems." These illustrations are given as themes for oral, blackboard, and slate exercises, in making original number-stories like those in the text.
- *Lessons XI and XII. Ten is here dealt with as a unit, that the child may get, at the outset, a correct view of our decimal notation.
- *Lesson XIII. The first of a series of test-lessons, and which are, therefore, purposely miscellaneous in character.
- *Lesson XV. The purpose of this lesson is to picture out the real meaning of "eleven, twelve," etc., and make the child realize it when counting.
- *Lessons XX to XXIII, and also Lesson XXXV. A summary and review of the numbers within ten, analyzed according to the Grube method.
- *Lessons XXXII and XXXIII may be either dictated by the teacher, or used as a silent desk exercise.
- Part ///.—*Lessons IV, VIII, XV, XIX, XXII, and XXV, constitute the groundwork of all operations within the hundred; each one introduces a new step, and should be thoroughly practiced with objects.
- *Lessons XXIV to XXXII embrace a complete review of numbers from ten to twenty, which should now be thoroughly practiced.
- Part /V.—* Lesson IV. In all cases of successive additions, subtractions, etc., state results, not the operations that produce them.
- *Lessons V, VII, IX, XII, XVI, XXII, XXIX, and XXX, each introduces a new step, and it is especially recommended that objects be used for illustration until the pupils have thoroughly mastered the subject.
- *Lesson XV. In this, as in all the lessons on fractions, pupils should be required to draw squares, and actually perform the operations with them before expressing in figures.
- *Lessons XX and XXIV. The large numbers given here are for practice exercises only—concrete applications in them would be inadvisable.
- *Lesson XXV. It is recommended that the teacher herself illustrate each step of this lesson with objects.

PART II.

READING LESSONS IN NUMBER,

AND DICTATION EXERCISES.

Combinations from One to Ten.

T	F	S	S	O	N	I.
_	ند،	J	J	v		

ADDITION.

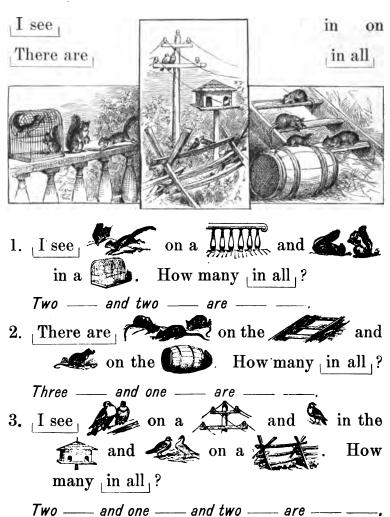
Phrase and words to be learned by sight.

and are

How many

Ь	
1.	How many are and ??
	One and one are two,
2 .	How many are and?
	Two and one are
3.	How many are and and??
	One — and one — are — — are — —.
4.	How many are and and and ??
	Two and one and one are

Phrases and words to be learned by sight.



Words to be learned by sight and sound.

bÿ	ŭn'der	ănd	€ăts
F		合意	
			Strangeled and Strang
1.	I see in a a . How many		in
2 .	Three and two a There are in a	and	by
2	the How many Three — and three — I see in the	are	- <u>'</u>
J.	the and How many in all ?		
	Two and two and	two are _	

SUBTRACTION.

are left	lĕss
will be left	thēşe
take away fly away h	op away
1. If I take away two of these how many will be left?	W.
Three less two are	
Four —— less two —— are —— 3. Take away one of these many are left?	··. How
Five — less one — are — hop as two fly away, how many will five — less two — less two — is	be left ?

were there	lěft	răn
ran away	lămb	ş ōn'ly
A	ran away an How many we	
lambs less three 2. There were only two. How is	Now t	here are
Six less 3. There are will b	. If three f	y away ,
Six less three 4. If I _take away t and you _take a	two of these	
will be left? Six —— less two ——	less two are -	

Here are roll away	frŏġş
of them swim away	\mathbf{bees}
out of runs away	gōat
1. Here are If o	one goat
runs away, how many will be le	ft _?
Six goats less one goat are goats.	
2. If two of these run aw	vay and
you take away one, how	many
will be left ?	•
Six — less two — less one — are —	
3. There are . If four	of them
hop away, how many will be lef	<u>t</u> _?
Six frogs less four frogs are —— frogs.	
4. Here are . If two run av	vay and
three swim away, how many will	be left?
Seven — less two — less three — are -	

5 .	Eight bees are on this . If two bees
	fly away, how many will be left?
	Eight bees less two bees are bees.
6 .	If you take Juli out of this est,
	how many will be left?
	Eight — less three — are —
7.	If six of these swim away,
	how many will be left?
	Nine —— less six —— are —— ——.
8.	Here are . If five of them roll away,
	how many will be left?
	Ten less five are

^{*} Making up and writing out original problems.



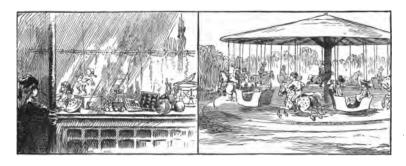
LESSON VII.

MULTIPLICATION AND DIVISION.

, ,	-	
How many times	lĕgş	thĭs
Are any left	bŏx	pĭnks
can you find	$\mathbf{t}reve{\mathbf{o}}\mathbf{p}\mathbf{s}$	kītes
1. How many legs have these		<u>.</u> ?
Two times two legs are leg 2. How many times M M have t	AEA J	
How many in all? Three times N are		
3. How many times can you tops from these will be left?	take awa ? How	لست
From five tops you can take and — top will be left.	Q Q —	_ times,
4. In these two kites can you find?	now many	times
In six kitos there are time		

5 .	From these how many times can you
	take two pinks? Are any left?
	From seven pinks you can take 🔻 — times,
	and — pink will be left.
6 .	How many times four legs have
	How many legs in all?
	Two times — legs are — legs.
7.	How many times III in this ??
	How many in all?
	There are times in the box.
	times two are

Making up and writing out original problems.



LESSON VIII.

Words to be learned by sight and by sound.

ne	r	appies	irom	buaş
toe	es ·	fingers	ĕggş	nŭts
			~~	
1.		w many times can you take		out of
	\mathbf{t}	his box Are an	y left _. ?	
	Fron	m six — you can take 👊	$\mathbf{L} = t$	imes.
2.	Ho	w many times five fingers or	$_{ m these}$	3 445
	Ī	How many fingers in all?		
	Ther	re are —— fingers in all. I	'n	fingers
	t	here are —— times five finger	8.	
3.	Hov	w many toes has a cat on her	r	?
	Two	times five toes are toes.		
4 .	Hov	w many toes has a cat on her		?
		times — toes are — toes		
5 .	Hov	w many times can you take		from
	t	hese ?	•	
	Fron	n eight buds you can take	7	times.

6. How many times three nuts are there on this How many in all?

There are — times three nuts. Three times are — nuts.

7. How many times can you take four apples from these Are any left?

From nine apples you can take times, and — apple will be left.

8. How many times two eggs can you take from this ?

From ten eggs you can take times.

^{*} Making up and writing out original problems.



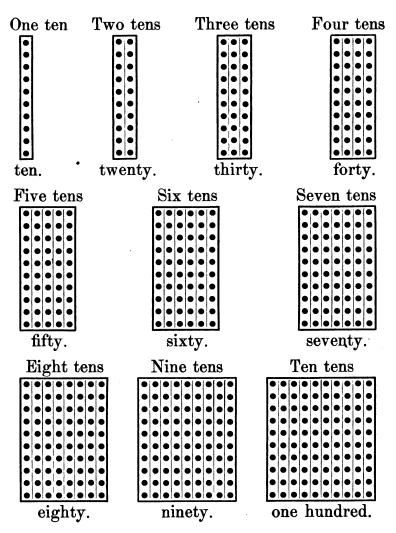
*LESSON IX.

0	ne half one fo	ourth	mīne
01	ne third one fi	${f fth}_{oldsymbol{oldsymbol{oldsymbol{h}}}}$	shĕllş
as	s many she ga	ave	e ătch
1.	Mark has	, but I have only	one half
	as many. H	ow many have I?	
	One half of two —	— <i>i</i> s ——,	
2.	I have	but Carl has only	one third
	as many . H	ow many has Carl	?
	One third of three	<i>i</i> s,	
3.	What is one four	th of E ?	
	One fourth of four	apples is —— —	
4.	If you catch one	fifth of these	, how
	many will you	have ?	,
	One fifth of five —	<i>i</i> s,	,
5 .	What is one half	of 420	?
	One half of six —		

6 .	May had She gave one third of
	them to Ned. How many did he get?
	One third of six — are —
7.	If I give one half of these to you, how
	many will you have?
	One half of four —— are —— ——.
8.	If I take away one half of these
	how many will be left?
	One half of eight —— are —— ——.
9.	One fourth of these is mine.
	How many shells have I?
	One fourth of eight shells are shells.
10.	Ann had She gave one third
	of them to me. How many did I get?
	One third of nine — are — .
11.	If you take one half of these
	how many eggs will you have?
	One half of ten eggs are —— eggs.

	Phrases to be learned by sight and words in	by sound.	
n	ore than have I	lŏg	pŏnd
1.	Carl has and I have	Ø .	How
	many more has Carl than I?		
2.	Nellie has and I have	_	How
	many more have I than Nellie	?	
3.	May has and Ned has		How
	many more has May than Ned	?	
4.	Five are more than two Here are In the		there
	are five frogs. How many mo		
	there on the log than in the	•	-
	Six frogs are more than five f	rogs.	
5 .	I have and May	has §	%.
	How many more have I than	May?	
	Eight are more than tw	o ——.	

COUNTING BY TENS.



DICTATION EXERCISES, WORKING WITH TENS.

Phrases to be learned by sight and words by sound.

Give the name like these add make have you made enough more e-rase made

- 1. Make two tens like these and name them.
- 2. Make four tens. Give the name.
- 3. Make three tens. Name them. Add enough more tens to make five tens.
- 4. Make six tens. Name them. Erase three tens. Name the tens you have left.
- 5. Make three tens and four more tens. How many tens in all? Give the name.
- 6. Make eight tens. Name them. Erase half of the eight tens. How many are left?
- 7. Make five tens. Name them. Add enough more tens to have nine tens.
- 8. Make nine tens. Name them. How many times can you take three tens from ninety?

- 9. Make five tens. Name them. Add five more tens. How many tens in all have you?

 How many times have you made five tens?

 Give the name of two times five tens.
- 10. Make three tens. Name them. Add three more tens. How many tens have you now? Name? Add three more tens. How many times have you made three tens (thirty)? How many tens have you in all? Name?
- 11. Make two tens. Name them. Add two more tens (twenty). How many tens have you?

 Name? Add two more tens. How many tens have you now? Name? Add two more tens. How many now? Name?

 Add two more tens. How many times have you made two tens? How many tens in all? Name?

MISCELLANEOUS EXAMPLES.

Phrases to be learned by sight and words by sound.

Picture it has he left shall sell with counters on your slate slate dots

- 1. How many are and and and?
- 2. Ned had and and Four of them ran away. How many has he left?
- 3. I have two times and more.

 How many have I in all? Picture it with counters or with dots on your slate.
- 4. I have three times and two times How many in all? Picture it.
- 5. How many times in this ?
- 6. Here are and and If I take four, how many will be left? Picture it.
- 7. I have four times . If I sell five of them, how many shall I have left?
- 8. How many are three times three kites?

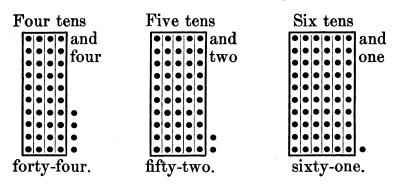
the more to which much cents
the more money more ap'ples

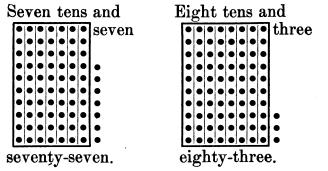
- 1. How many more legs has a 🎉 than an
- 2. Carl has and May has . Which has the more money ?
- 3. Nita has . How much more money has she than Carl? How much more than May?
- 4. How many more legs has a * than a ??
- 5. I gave to Jamie and to Mary. To which did I give the more?
- 6. Nellie found and Jack found only three shells. How many more shells has Nellie than Jack?
- 7. I have seven cents and Jamie has nine cents.

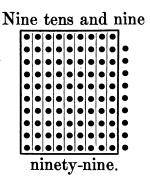
 Which of us has the more money? How much more?

COUNTING BY ONES, ABOVE TEN.

One ten and one eleven.	One ten and two two twelve.	One ten and three thirteen.	One ten and four four fourteen.
One ten and five fifteen.	One ten and six sixteen.	One ten and seven seven seventeen.	One ten and eight eight eighteen.
One ten and nine nine nineteen.	Two tens	Two tens and six twenty-six.	Three tens o and five five thirty-five.

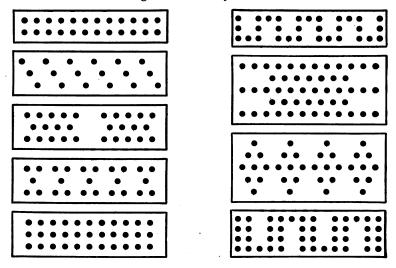






GATHERING INTO TENS.

The pupils to be directed to copy these groups of dots, putting the tens into boxes, and to tell or write in words how many tens and how many single dots over in each group, and to give the name of the number.



The pupils to be directed to make as many dots as these numbers call for, and to put the tens into boxes.

thirteen	fifteen	nineteen
thirty-one	fifty-eight	ninety
fifty-seven	seventy-two	sixty-three
sixty-five	ninety-nine	seventy-five
eighty-nine	forty-six	eighty-two

Words to be learned by sound.

twice

found

dāiş'ieş

dăn'de-lī-onş

- 1. Carl has in the and May has and Nita has . How much have they in all?
- 2. I found dandelions. I gave to May and I have left. How many had I in all?
- 3. I have and Mary has twice as many.

 How many has Mary?
- 4. Jack has Ned? How many have they in all? Picture it.
- 5. If you give me half as many daisies as are in this , how many shall I have?
- 6. If you give me twice as many daisies as are in this , how many shall I have?

Put into twice as many bunch take out of half of the rest wheels

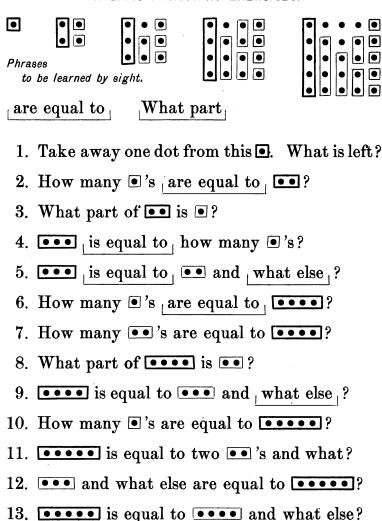
- 1. Will gave these to me. He gave twice as many to Carl. How many shells did he give to Carl? Picture it.
- 2. Rose had half of the rest. How many has she left?
- 3. Tom had we have the state one bunch and gave one third of the rest to Ned. How many has he left? Picture it.
- 4. How many times can you put into this box?
- 5. How many times a can you take out of this box?
- 6. How many wheels are there in five times ? Picture it.

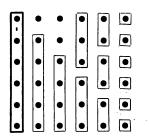
can I buy for kīte pāy çĕnts What change thăt pĕn'çil €ŏst 1. I gave 5 for . How much is that for one pencil? 2. How many times _____ can I buy for ? 3. How many times can I buy for can I buy for 4. If costs , what will two tops cost? 🚺 cost 🥵, how many can you get for ? Picture it. 6. If cost in how many can you get for a dime? What change over? 7. A costs . Ned has and . How many more cents must he get to buy a kite?

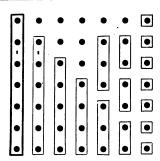
8. Alice had (5) and (11). She gave six cents for a (2). How many cents has she now?

*LESSON XX.

ANALYTIC MEASURING EXERCISES.

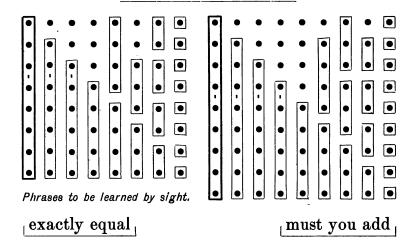






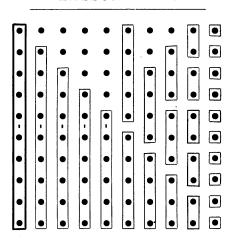
like numbers

- 1. $\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$ is equal to five and what else?
- 2. $\bullet \bullet \bullet \bullet$ and what are equal to $\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$?
- 3. What two <u>like numbers</u> are equal to What three like numbers?
- 4. What part of ••••• is •••? Is ••?
- 5. •••••• is equal to ••••• and what?
- 6. How many dots must you add to ••••• to make it equal to •••••?
- 7. Four and what else are equal to ••••••?
- 8. Two ••• 's and what are equal to seven?
- 9. ••••• is equal to three ••'s and what?
- 10. What part of $\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$ is \bullet ?



- 1. What must you add to to make it equal to •••••• ?
- 2. How many ••'s are exactly equal to
- 3. What part of •••••• is ••?
- 4. What other like numbers are exactly equal to ••••••?
- 5. How many more dots in •••••• than in •••••?
- 6. What part of $\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$ is $\bullet \bullet \bullet \bullet \bullet$?

- 7. ••••• is equal to two ••• 's and what else?
- 8. How many more dots in •••••• than in •••••• ?
- 9. •••••• and how many are equal to
- 10. How many dots must you add to •••••••• to make it equal to •••••••?
- 11. $\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$ is equal to six and what?
- 12. •••••• is how many more dots than
- 13. Two ••••'s and what else are equal to
- 14. Find all the like numbers that are exactly equal to ••••••?
- 15. $\bullet \bullet \bullet$ is what part of $\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$?
- 16. How many ••'s and what else do you find in •••••••?



- 1. $\bullet \bullet \bullet$ is equal to nine and what?
- 2. What must you add to •••••• to make it equal to ten?
- 3. Ten is how many more than •••••• ?
- 4. ••••• and what are equal to ten?
- 5. How many •••••'s are equal to ten? How many ••'s? How many •'s?
- 6. What part of ten is ••? Is ••••? Is •?
- 7. Two ••••'s and what are equal to ten?
- 8. Three •••'s and what are equal to ten?

must you get spěnd dīmeş spěnt piēç'eş

- 1. How many pieces can I get in change for a ?
- 2. How many pieces can I get in change for What else?
- 3. How many pieces can I get in change for two dimes?
- 4. Jack had and and and . He spent four cents for a and two cents for a How many cents in all did he spend? How many cents has he left?
- 5. How many can you get for ??
- 6. If I give you to buy a must you get ?
- 7. How many can you get for ??

LESSON XXV.

EXERCISES IN EQUAL AND UNEQUAL NUMBERS.

Phrases to be learned by sight and words by sound.

equal groups	each group	ŭn'der
all the sets	write about	be-sīde'

- 1. Make ten dots in five equal groups.
- 2. Beside <u>each group</u> write the name of the number of dots in it.
- 3. Under the last group write the name of the number of dots in | all the groups |.
- 4. In the same way make and write about two more sets of equal numbers in ten.
- 5. In the same way make and write about all the sets, of equal groups in nine dots. In eight dots. In six dots. In four dots. In seven dots. In five dots. In three dots. In two dots.

ten dots.

sever

Phrases to be learned by sight.

 unequal groups
 both groups
 nāme

 the other sets
 the same way
 nŭm'ber

- 1. Make ten dots in two unequal groups.
- 2. Beside each group write the name of the number of dots in it.
- 3. Under the last group write the name of the number of dots in both groups.
- 4. In the same way make and write ton dots.

 about all the other sets of two unequal groups you find in ten dots.
- 5. In the same way make and write about all the sets of two unequal groups you can find in nine dots. In eight dots. In seven dots. In six dots. In five dots. In four dots. In three dots.

LESSON XXVII.

Words to be learned by sound.

ŏr'anġe

€ŭt

shâre

mĕl'on

ĭn'to

sup-pōşe'

- 1. Into how many parts is this orange cut?
- 2. What is the name of one part?
- 3. If I give you one half, how much of the orange will be left?
- 4. May, Carl, Nita and Ned are to have this melon. Into how many parts must I cut it so that they will have equal shares?
- 5. This is Ned's share . What part of the melon is it? What part will be Nita's share? Carl's share? May's?
- 6. If Carl gives his share to May, how much will she have then? Suppose Ned gives May his share, too. How much will she have?

LESSON XXVIII.

Phrases to be learned by sight and words by sound.

equal parts breāk eāke
my part each part eăn'dy eāne

- 1. Break this candy cane into three equal parts. What is the name of each part?
- 2. If you give me one third of it and Will one third, how much will you have left?
- 3. If I give my part to Will, how much will he have then?
- 4. Cut this cake into two equal parts. Name the parts.
- 5. Cut each half into three equal parts. How many equal parts in all are there?
- 6. What is the name of each of these parts?
- 7. To how many boys can you give one sixth each and have one sixth left?

Words to be learned by sound.

flŏck

fĭsh

find

e-rāse'

grāpes

- 1. If you catch one of these part of the flock will you have?
- 2. If you give one of these what part of all the grapes shall I have?
- 3. Here are that Carl and May found.

 May found one nut and Carl found seven.

 What part of them did May find? Carl?
- 5. Make ten dots. Erase one dot. What part of all did you erase? What part is left?
- 6. Make ten dots in twos. How many twos are there? Erase one two. What part did you erase? What part is left?

Which of us pāy flăg ēach thus piēç'es

- 1. Carl has , May has two pieces, Will has ten pieces and I have three pieces and a piece. Which of us has the most money?
- 2. Jamie has . He paid . He paid . He pay for all of them? Picture it—thus,
- 3. If a costs two , how much will three sleds cost? Picture it.
- 5. If a costs five dimes, how many can I get for ten dimes?

Phrases to be learned by sight and words by sound.

in the middle still longer line

- 1. Make a line four times _____one inch__ long.

 How long is it? Rub out half of it.

 How long is it now?
- 2. Make a line six inches long. Put a dot at each end. Put a dot in the middle. How many times three inches long is it?
- ach end. Make this same line two inches longer. Put a dot at the end.

 Now make it two inches still longer, and put a dot at the end. How many inches long is your line now? How many times two inches long is it? How long is one third of it?

LESSON XXXII.

* DICTATION EXERCISES ON PARTS.

Phrases to be learned by sight and words by sound.

the other half splint pā'per

- 1. Break a match-stick into two equal parts.

 Name the parts.
- 2. Give away one half. What have you left?
- 3. Give away the other half. What have you left now?
- 4. How many times did you give away one half?
- 5. Cut a slip of paper into three equal parts.

 Name the parts.
- 6. Give one third to Jennie. How many thirds have you left?
- 7. Give another third to Jennie. How many times have you given away one third? How many more thirds have you to give away?

LESSON XXXIII.

Phrases to be learned by sight and words by sound.

Can you tell way ap'ple
Who can tell why eas-i-est

- 1. Cut an apple into two equal parts. Name the parts. Cut each half into two equal parts. How many parts have you now?
- 2. Can you tell why they are called fourths?
- 3. To how many boys can you give one fourth each?
- 4. To how many can you give two fourths each?
- 5. Give away three fourths of your apple.
 What have you left?
- 6. Cut a slip of paper into eight equal parts.

 Who can tell the easiest way to do it?
- 7. Group your eighths by twos. How many times two eighths have you? To how many boys can you give two eighths each?

- 8. Group your eighths by fours. How many four eighths have you?
- 9. If you give me half of all the parts, what will you have left?
- 10. If you give me five eighths, what will you have left? If you give me seven eighths?

 Two eighths? Three eighths?
- 11. Break a match-stick into three equal parts, and break each of these parts in half.

 How many parts have you? Name them.
- 12. Can you tell why they are called sixths?
- 13. If you give two sixths to me and two sixths to Carl, what will you have left?
- 14. If you give away half of all the parts, what will you have left?
- 15. If I give you five sixths of an orange and Will gives you one sixth of an orange, what will you have?

*LESSON XXXIV.

Phrases to be learned by sight and words by sound.

Draw a line řdý'eş ăn'gleş
Are they equal fāç'eş côr'nerş

- 1. How many angles has this $tr\bar{t}'$ angle? How many sides has it?
- 2. Make two triangles. How many angles have two triangles? How many sides?
- 3. Make three triangles. What can you tell about them?
- 4. How many sides has this square? How many corners has it?
- 5. Make two squares. How many sides have two squares? How many corners?
- 6. How many sides has this ? How many corners has it? pěnťa-gon
- 7. Make two pentagons. Tell all you can about them.

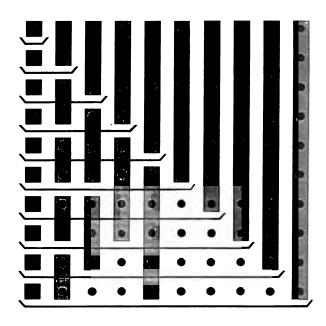
•
8. How many faces has this ? How
many edges has it? $e^{\bar{u}b\bar{\theta}}$
9. Make two eūbes and count the faces. The
edges. The corners.
10. Make a square like this Draw a
line from (a) to (b). How many triangles
have you? Are they equal?
11. Draw a line from (c) to (d). How many tri-
angles have you now? Are these equal?
12. What part of the square is each triangle?
What else can you tell about them?a_
What else can you tell about them? 13. Make another square like this Draw a line from the and then one from
Draw a line from (a) to (b) and then one from
(a) to (c). How many triangles have you here?
Are they equal? Draw a line from (a) to (a).
How many triangles have you now? Are
these equal? What part of the square is
each triangle? Tell all you can about them.

TABULATED REVIEW

OF ALL COMBINATIONS IN EACH NUMBER FROM ONE TO TEN.

Each number being designated in turn, the pupils are directed to find and tell:

- I. All the possible additions which make the number.
- 2. All the subtractions which can be made from the number.
- 3. All the divisions of the number which can be made, both without and with remainders.
- 4. All the multiplications which form the number.



ORIGINAL ORAL PROBLEMS.

Each pupil may be permitted to choose a section, and to tell all the combinations he can find within its limits.

PART III.

Combinations in Numbers from Ten to Twenty.

LESSON I.

LEARNING THE FIGURES FOR NUMBERS FROM ONE TO TEN.

Words to be learned by sound.

groups năm'		berş	pāge	$\mathbf{sh}\mathbf{\bar{o}w}$	е ŏр ′ у		
1.	1. $\underbrace{\begin{array}{c} \bullet \\ \text{one} \\ 1 \end{array}}_{\text{two}} \underbrace{\begin{array}{c} \bullet \bullet \\ \text{two} \\ 2 \end{array}}$		three 3	four 4	$\begin{array}{c} \bullet \bullet \bullet \\ \bullet \bullet \bullet \\ \hline \text{five} \\ 5 \end{array}$	six 6	
	sev	en 7	eight 8	nin		ten 10	

- 2. What does the word under each group of dots tell? The figure under each group tells the same thing.

- 4. Make the groups of dots these figures call for: 3, 7, 4, 5, 2, 8, 6, 1, 9.
- 5. Put ten sticks or other counters on your desk. Tie them into a ten. Can you tell by figures how many you have? Will it do to write only the figure 1 to show that you have one ten?
- 6. How many counters does 1 stand for? How, then, can you make the figure 1 stand for one ten?
 - The I must be put in the ten's place, that is, in the second place to the left, to show that it stands for one bundle of ten counters, and not for one single counter.
- 7. Which of these figures, 10, tells the number of tens? Which figure tells that there are no ones?
- 8. Make the figures which stand for eight ones; eight tens; six ones; six tens; five ones; five tens; nine ones; nine tens; three ones; three tens.
- 9. Write the figures for six, four, nine, eight, three, seven, two, five, one, ten.
- 10. Copy the groups of dots on page 43, and under the names of the numbers write the figures which tell the numbers.

LEARNING THE FIGURES FOR NUMBERS FROM ONE TEN TO TWO TENS.

Phrases to be learned by sight and words by sound.

learn ex-aet'ly Work out stands for

- 1. You have now learned the figures that stand for all numbers of things from one thing to ten things.
- 2. You have also learned that we count the tens just as we did the ones—thus: one ten (ten), two tens (twenty), three tens (thirty), etc.
- 3. And also that we use exactly the same figures for tens as for ones, the only difference being that, when a figure stands alone, it means that number of *ones*, and that, when it stands in the second place to the left, it means that number of *tens*.
- 4. We have now to learn to count and make the figures by "tens and ones." If we have this many counters, we say we have sixteen, meaning six single counters and one ten—six and ten, sixteen.
- 5. If we have one ten and two single counters, we say we have twelve, a word which means "two and ten."

- 6. Thirteen means three and ten; fourteen, four and ten; fifteen, five and ten; seventeen, seven and ten; eighteen, eight and ten; nineteen, nine and ten.
- 7. Copy the groups of dots on page 48, and under the names of the numbers write the figures which tell how many tens and how many ones in each group.
- 8. Put exactly twenty counters on your desk.

 Now count out ten and tie in a bundle.

 What do we call this bundle? (A ten.)
- 9. Put two single counters to the right of this ten. How many in all? Tell by figures how many tens and how many ones.
- 10. Work out with your counters and tell first by words and then by figures how many are 1 ten and 5 ones; 1 ten and 7 ones; 1 ten and 4 ones; 1 ten and 8 ones; 1 ten and 3 ones; 1 ten and 6 ones; 1 ten and 9 ones.
- 11. Tie the single counters you have been using into a ten. How many tens have you now? Have you any single counters left over?
- 12. What is the name of two tens? Write twenty in figures. What figure do you use to show the number of tens? What figure do you use to show that there are no ones?

- 13. What number of counters is 1 more than 10? What number is 1 more than 13? Than 15? Than 17? Than 14? Than 12?
- 14. What number of counters is 1 less than 20? What number is 1 less than 16? Than 18?
- 15. Group your counters as these figures tell you, and write the names of the numbers. 13, 16, 18, 12, 15, 17, 11, 14, 20, 19.

APPLICATIONS.

- 1. Which would you rather have, 8 pennies or 80 pennies? 9 dollars or 30 dollars? One dime or nine cents? Why?
- 2. In 19, which figure stands for the greater number of things, the 1 or the 9? Why?
- 3. Would you rather have the number of tardy marks the 1 stands for, or the number the 9 stands for? Why?
- 4. Nat and Will have 17 nuts. Nat has as many as the 1 stands for, and Will as many as the 7 stands for. Which has the greater number of nuts?
- 5. In 11 the figures are alike. Do they each stand for the same number of things? Why not? What is the difference?

LEARNING THE SIGNS + AND =. Words to be learned by sound.

shôrt'er	sīgn	fĭg'ureş
wrīt'ing	wõrd	sĕn'ten-çeş
1. How many are	and	?
Four	and five are	nine.
2. Here is a sh	orter way	of writing this:
4+5=9.		
3. What word is	the figure 4	used for?
What word is	the sign +	used for?
What word is	the $sign = \frac{1}{2}$	used for?
4. Write these se	entences and	put figures and
signs in place	e of the word	s and the dashes:
Three and five	are	
Seven and two	are	
Nine and one	and five are	
Seven and thre	e and six a	re —.
Four and six a	and eight are	e
	•	and one are —.
Six and four a	nd two and	three are —.
Five and six a	nd seven and	l one are —.
Eight and two	and two an	d two are ——.
_		nd three are —.
Six and three	and one and	eight are —.

ADDITIONS BETWEEN 10 AND 20. Phrase to be learned by sight and words by sound.

all the additions,

sĭn'gle

sīgnş

1. Put 2 single counters beside a ten, thus:
In which place have you put the ten?
In which the single counters?



- 2. What number is this? Tell by figures.
- 3. Add three counters to your 12 counters, then tell by figures and signs how many are 12 counters and 3 counters. Let the letter c. stand for counters.

Thus: 12 c. + 3 c. = 15 c.

- 4. Work out with your counters and tell in the same way how many are 12 c. and 4 c.; 12 c. and 6 c.; 12 c. and 8 c.; 12 c. and 5 c.; 12 c. and 7 c.; 12 c. and 1 c.
- 5. Make a 13 with your counters. Add 7 counters. Tell by figures and signs, thus: 13+7=20. Tell in the same way how many are 13 and four; 13 and six; thirteen and 5; thirteen and three.
- 6. Make a 14 with your counters, and work out with the counters you have left over and tell by figures and signs all the additions you can make.

- 7. Work out with your counters and tell by figures and signs how many you must add to 15 to have 18; how many you add to 15 to have 20; to have 17; to have 19.
- 8. In the same way find and tell how many you must add to 16 to have 20; to have 18; to have 19; to have 16. How many must you add to 17 to have 20? How many must you add to 18 to have 20?

LESSON V.

CONCRETE APPLICATIONS.

- 1. Jane has a dozen eggs. How many more must she buy to have 16?
- 2. I rode 14 miles in the train and 4 miles in the stage-coach. How far did I go?
- 3. May read 15 pages in her reader last week, and she has read 4 to-day. How many pages in all has she read?
- 4. I spent 13 cents for a kite, and 4 cents for a string, and had one cent left. How many had I at first?
- 5 Carl has 16 cents in his bank and 3 cents in his pocket. How much money has he?

- 6. There are 11 goats on the hill-side and 5 goats by the road-side. How many in all?
- 7. How many are 13 lilies and 3 lilies?
- 8. There are 12 nuts on the tree and 5 nuts on the ground. How many altogether?
- 9. On one branch I can count 14 bees and on another 5 bees. How many on both?
- 10. How much money is 11 dollars and 7 dollars?
- 11. Carl has 15 doves and May has 3. How many doves have they both?
- 12. I have 14 buttons, but I need to have 17. How many more must I get?
- 13. I can see 12 ships sailing down the bay and 7 ships sailing up the bay. How many in all do I see?
- 14. How many are 15 flags and 4 flags?
- 15. In the pond there are 11 frogs and on the bank there are 3. How many in all?
- 16. I have made 17 marks on my slate and have 3 more to make. How many shall I have made then?
- 17. How many are 13 cherries and 6 cherries?
- 18. I have a dozen marbles, Nat has half a dozen, and Will has 2. How many have we altogether?

OBJECT AND SLATE WORK.

Work out with counters, copy and complete.

$$12+3 = 15+2 = 14+3 = 17+2 = 13+2 = 12+5 = 13+4 = 12+7 = 14+5 = 13+6 = 16+1 = 14+2 = 15+4 = 16+3 = 11+6 = 12+4 = 14+1$$

Copy, complete, and afterward prove with counters. Try to do them in five minutes.

$$1+4=$$
 $4+5=$ $1+7=$ $3+5=$ $11+4=$ $14+5=$ $11+7=$ $13+5=$ $4+1=$ $5+4=$ $7+1=$ $5+3=$ $14+1=$ $15+4=$ $17+1=$ $15+3=$

*ORIGINAL PROBLEMS.

Write or tell number stories about anything you like, using:

RAPID ORAL CALCULATIONS.

Read off quickly, giving the answers at once.

12 + 2	13 + 3	14 + 4	15 + 5	-11 + 1
3 + 16	4 + 11	5 + 12	17 + 3	19 + 1
10 + 4	5 + 13	16 + 4	18 + 2	13 + 7
5 + 14	15 + 4	8 + 11	13 + 6	17 + 2

LEARNING THE SIGN —, Words to be learned by sound.

mõre	thăn	lĕss	ex-prĕss'	dăsh'eş
1. How	many are	• • •	less three	dots?
-	Eight	less three	are five.	
2. $8 - 3$	=5 is a s	shorter w	ay of writi	ng this.
3. What	t word is	the sign	used for	?
4. May	has this	many pe	ncils, IIII,	and Carl
ha	s this mai	ny, II. I	How many r	nore pen-

cils has May than Carl?

Five is three more than two.

- 5. 5-2=3 is the way we express this in figures and signs.
- 4. Write these sentences with figures and signs in place of the words and dashes:
 - Seven less five are —. Ten is more than six. Sixteen less six are —. Nine is more than seven. Twelve less two are —. Seventeen is more than seven. Eighteen less nine are —. Nineteen is more than one.
- 7. How many are 14 less 3? 16-4=? 17-5=?
- 8. How many more are 13 than 10? 15-3=? 18-5=? 19-9=? 16-3=?

SUBTRACTIONS BETWEEN 10 AND 20. Phrase to be learned by sight and words by sound.

all the subtractions un-ty'ing fig'ures with-out' count'ers

- 1. Put one bundle of ten and ten single counters on your desk. How many in all are there?
- 2. From your 20 counters take 8 counters, and tell by figures and signs how many are left. Thus: 20 c. -8 c. = 12 c.
- 3. In the same way find and tell how many are 20 c. less 6 c.; 20 c. less 3 c.; 20 c. less 5 c.; 20 c. less 9 c.; 20 c. less 7 c.; 20 c. less 2 c.; 20 c. less 4 c.; 20 c. less 1 c.
- 4. Have 18 counters on your desk. Now find and tell how many are 18 counters less 8 counters; 18 less 6; 18 less 4; 18 less 2; 18 less 7; 18 less 5; 18 less 3.
- 5. Have 17 counters on your desk. Find and tell all the subtractions you can make without untying the ten bundle.
- 6. Have only 16 counters on your desk. In the same way as before, find and tell all the subtractions you can make. Do the same thing with 15 counters; with 14; with 13; with 12; with 11.

CONCRETE APPLICATIONS.

Phrases to be learned by sight and words by sound.

this morning	āte	$\mathbf{br\"{o}ke}$	${f sehoo}$
this afternoon	ēat	brōk' <i>e</i> n	stā′bl€

- 1. May had two dimes. She spent one cent for a paper doll. How much money had she left?
- 2. Carl, too, had two dimes, and he spent { cents for a top. How much had he left
- 3. After Carl had bought his top he lost 4 of hi cents, and then he put what he had lef into his bank. How much did he put into his bank?
- 4. May gave the rest of her money, all but 6 cents, to Will and Nita. How much did she give away?
- 5. Nell had a dozen and a half of eggs in a basket. She let it fall and broke 7 of the eggs. How many were not broken?
- 6. 17 goats less 6 goats are how many?
- 7. On his way to school Tom found 14 nuts. He ate all but three. How many did he eat:
- 8. This morning there were 19 cows in the stable; now there are only 15. How many have been taken out?

- 9. This afternoon Jack had 16 bags of pop-corn to sell. He sold only five. How many has he left?
- 10. Jennie's little white hen sat upon 15 eggs.
 All hatched out but 2. How many little chickens has she?
- 11. This afternoon Will and Walter found 19 pond-lilies. Will found 8 of them. How many did Walter find?
- 12. There are 18 trees in a field. 5 are appletrees and the rest are plum-trees. How many plum-trees are there?
- 13. John had 17 marbles and lost 4. How many has he left?
- 14. Nat is 13 years old and Will is 11. How much older is Nat than Will?
- 15. May has 16 buttons and Nita has 13. How many more has May than Nita? Picture it, thus:

16 is 3 more than 13.

OBJECT AND SLATE WORK.

Work out with your counters, copy and complete.

$$20-7 = 19-9 = 13-1 = 11-1 = 17-3 = 14-2 = 12-2 = 18-4 = 12$$

Picture these with dots (as in example 15, page 86) and copy, giving the answers.

How many more are

* Copy, complete, and prove with counters.

$$18-?=6$$
 $20-?=7$ $14-3=?$ $15-?=12$ $17-6=?$ $?-17=2$ $?-7=11$ $13-3=?$

RAPID ORAL CALCULATIONS.

Read and give the answers at once.

$$10 \text{ less } 4=?$$
 $9 \text{ less } 3=?$ $6 \text{ less } 4=?$ $13 \text{ less } 2=?$ $20-4=?$ $19-3=?$ $16-4=?$ $18-8=?$ $11-1=?$ $15-1=?$ $20-8=?$ $19-5=?$

What is the difference between

MAKING UP TENS.

- 2. Make a ten of the first group by adding counters from the second group; thus:
- 3. How many counters did you put to the seven to make ten? Then how many of the 5 are left?
- 4. Show by figures what you have done; thus: 7+3+2=12. Then 7+5=?
- 5. Place your counters in groups of 6 and 9; thus: ||||||, |||||||. Make a ten of the six with counters taken from the nine; thus: ||||||||||||.
- 6. How many counters did you add to the 6 to make a 10? How many of the 9 are left?

 Then 6+4+5=? 6+9=?
- 7. Place your counters in groups as these figures tell you, then add, making a ten of each first group. Write every example both ways; thus: 7+3+3=13. 7+6=13.

CONCRETE APPLICATIONS. Words to be learned by sound.

bōth	pâir	$\mathbf{g}\mathbf{\ddot{a}r'}\mathbf{d}e\mathbf{n}$	write	üşed
a-gō'	shoeş	rōṣe'bŭsh	wrōte	doz'en

- 1. In my garden there are 6 roses on one rosebush and 5 roses on another. How many roses are there on both bushes?
- 2. Carl wrote 8 words on his slate this morning and 4 this afternoon. How many in all did he write?
- 3. If there are 7 buttons on one of little Nell's shoes, how many are on the pair?
- 4. Nine roses and 5 roses are how many?
- 5. In one box there are 9 pencils and in the other box there are only 2. How many pencils in both boxes?
- 6. Jane used a half a dozen eggs this morning and a half a dozen this afternoon. How many in all has she used to-day?
- 7. There are 8 bees on the bee-hive and 7 bees on the stand. How many altogether?
- 8. Four years ago Ned was 9 years old. How old is he now?
- 9. If I buy 7 marbles and you buy 5, how many shall we both have?

Words to be learned by sound,

spī'der

fiēld

ăn-ŏth'er al-to-gĕth'er

- 1. Will gave me 8 daisies this morning and 5 this afternoon. How many daisies has he given me to-day?
- 2. How many are 9 cents and 6 cents?
- 3. Carl has drawn a line 7 inches long. How many inches must he add to make it 11 inches long?
- 4. Carl has drawn another line 9 inches long. How many inches must he add to make it a foot long?
- 5. How many legs has a fly? How many legs has a spider? How many legs have a spider and a fly together?
- 6. How many are 7 frogs and 6 frogs?
- 7. There are 9 lambs in one field and 8 lambs in another. How many in both?
- 8. If you have three cents and I give you 8 more, how many will you have?
- 9. Nine blocks and how many more are 16 blocks?
- 10. Eight dots and how many more are 16 dots.
- 11. If there are 9 buttons on one of your shoes, how many buttons are there on the pair?

OBJECT AND SLATE WORK.

Work out with counters, copy and complete.

$$7+5=?$$
 $8+5=?$ $9+9=?$ $?+6=11$ $9+?=11$ $?+7=14$ $8+?=14$ $?+5=14$ $8+8=?$ $9+7=?$ $8+?=15$ $9+?=16$

Copy, complete, and prove with counters.

ORIGINAL PROBLEMS.

Make as many problems as you can of these.

? + ? = 16	? + ? = 18	? + ? = 13
? + ? = 17	? + ? = 15	? + ? = 12
? + ? = 14	? + ? = 19	? + ? = 20

RAPID ORAL CALCULATIONS.

Read and give the answers at once.

10 + 10 =	8 + 5 =	4 + 9 =
8 + 8 =	6 + 9 =	7 + 9 =
7+7=	7 + 6 =	8 + 6 =
9 + 9 =	8 + 7 =	9 + 5 =

BREAKING UP TENS.

- 1. Place 13 counters on your desk, the 10 tied in a bundle and the 3 single sticks beside it. Take 9 counters from these. Can you take 9 counters from the 3 counters? How many of the 9 can you take? What must you do to get the rest of the 9? Then 13-3-6=? 13-9=?
- 2. Place your counters as these figures tell you (the ten tied always in a bundle). From each number take 6. Write your work both ways; thus: 15-5-1=9. 15-6=9. 12, 14, 11, 13.
- 3. Place your counters as these figures tell you, and from each number take 7. Write your work both ways. 16, 13, 12, 15, 11, 14.
- 4. From each of these numbers take 9. Write your work both ways. 17, 13, 16, 11, 16, 12, 15, 18, 14.
- 5. From each of these numbers take 8. Write your work both ways. 11, 12, 13, 14, 15, 16, 17.
- 6. From each of these numbers take 5. Write your work both ways. 15, 14, 13, 12, 11.

CONCRETE APPLICATIONS. Words to be learned by sound.

learn beans peas length dif'fer-ence

- 1. May had 13 buttons on a string. She lost off7. How many has she left?
- 2. Fred is 16 years old? How old was he 8 years ago?
- 3. My slate is 11 inches long and May's is only 8 inches long. What is the difference in length between them?
- 5. Nita has a dozen blocks and Will has half a dozen. How many more blocks has Nita than Will? Picture it, thus:

Nita's blocks

Will's blocks

12 are ____ more than 6.

- 6. Fred planted 11 beans in his garden, and 9 peas. How many more beans than peas did he plant? Picture it.
- 7. There are 15 birds in a tree by my window. If 8 fly away, how many will be left?

- 8. In the pencil-box there are 14 sharp pencils and 8 dull ones. How many more sharp pencils than dull ones?
- 9. I see 11 frogs on a log. If 6 hop away, how many will be left?
- 10. Carl had 13 words to learn. He has learned 8, how many has he yet to learn?
- 11. How many more are 15 rabbits than 9 rabbits?
- 12. I am 12 years old and Carl is 7. How much older am I than Carl?
- 13. May made 16 paper-dolls and gave 9 of them to Nita. How many did she keep?
- 14. Carl had 14 doves. 7 of them flew away. How many has he left?
- 15. Four months of this year are past. How many are still to come?
- 16. There were 11 ducks in the pond. 7 have come out. How many are still in the pond?
- 17. Fred made 14 kites. He sold 9 of them. How many has he left?
- 18. If Jane buys a dozen oranges and gives away 8, how many will she have left?
- 19. John had a dozen and a half pop-corn-balls. He sold 9. How many has he yet to sell?
- 20. How many are 13 apples less 9 apples?

OBJECT AND SLATE WORK.

Work out with counters, copy and complete.

$$11-6=$$
? ? $-6=6$ $17-9=$? ? $-8=8$ $13-?=6$ $16-9=$? ? $-9=5$ $12-9=$?

Picture with dots, as in Lesson XVI.

How many more are

Copy, complete, and afterward prove with counters.

$$15 - ? = 6$$
 $12 - 9 = ?$ $? - 9 = 9$ $11 - 8 = ?$ $? - 5 = 13$ $11 - 9 = ?$ $? - 7 = 7$

RAPID ORAL CALCULATIONS.

Read and give the answers at once.

How many are

$$10 \text{ less } 9?$$
 $16-6?$ $13-7?$ $15-7?$ $16-8?$ $11-9?$ $17-9?$ $13-9?$

What is the difference between

9	and	13	?	3	and	12?	14	and	6?
16	and	17	?	17	and	8?	18	and	9?

How many more are

15 t	han	8?	18	than	9?	16	than	9?
17 t	han	92	13	than	6?	17	than	82

MULTIPLICATION.

LEARNING THE SIGN X.

Words to be learned by sight and sound.

۲ī	m	ΔQ	
U	ш	UO.	

plāçe

an-oth'er

show'ing

- 1. May wrote two words this morning and two words this afternoon. How many times did she write two words? How many in all?

 Two times two are four.
- 2. $2 \times 2 = 4$ is the shorter way of writing this.
- 3. What word is this sign \times used for?
- 4. How many mittens are there in a pair? How many in four pairs?

Four twos are eight.

- 5. Four 2's = 8 is the shorter way of writing this.
- 6. Write these sentences with figures and signs in place of the words and dashes:

Three times three are —. Six twos are —.

Four times five are ——. Three fours are ——.

Two times four are —. Five threes are —.

Nine times two are —. Two eights are —.

Six times three are —. Four fives are —.

- 7. How many are 2×5 ? $6 \times 2 = ?$ $10 \times 1 = ?$ $2 \times 8 = ?$ $9 \times 2 = ?$ $6 \times 3 = ?$
- 8. How many are two fives? Three fours =?
 Two 8's =? Nine 1's =? Seven 2's =?

EXERCISES IN MULTIPLYING.

- 2. Find with your counters and tell by figures and signs how many are ten 2's. Three 2's. Five 2's. Seven 2's. Nine 2's. Two 2's. Four 2's. Six 2's. Eight 2's.
- 3. Group your 20 counters by 4's, thus: |||| |||| |||| |||| How many 4's are there?
- 4. How many are 5×4 ? 2×4 ? 4×4 ? 3×4 ?
- 5. Group your counters in 5's, thus: ||||| ||||| ||||| ||||| How many 5's are there?
- 6. How many are four 5's? Two 5's? Three 5's?
- 7. Group your 20 counters in 3's, thus: ||| ||| ||| there and what else?
- 8. Put aside the two counters left over. How many counters have you now? $6 \times 3 = ?$
- 9. How many are 4×3 ? 3×3 ? 5×3 ? 2×3 ?
- 10. Group your 20 counters by 6's. How many of them do you need to make even 6's? Put the others aside. How many 6's are there?
- 11. How many are two 6's? Three 6's?
- 12. Group your counters by 9's. How many of them do you need to make even 9's? How many are two 9's?

CONCRETE APPLICATIONS. Words to be learned by sound.

${f pr}{f ompt}$	ẽarn	ōld	lēast
rē'al-ly	lēarn	ōld'er	$\mathbf{m}ar{\mathbf{o}}\mathbf{s}\mathbf{t}$
brĕak'fast	vērse	$ar{ ext{old'est}}$	mönth

- 1. Every day for one week May was given 2 cents for being prompt at breakfast. How many times two cents had she at the end of the week. How many cents in all?
- 2. Carl was given 3 cents for every basketful of weeds he picked out of the garden. How much did he get for 5 basketfuls?
- 3. When eggs are 3 cents each how much will a half dozen cost?
- 4. May was to get 4 cents for every verse she learned. She learned one verse every week for a month. How much did she earn?
- 5. Carl, who is older than May, was to get 2 cents for every verse he learned. He learned 2 verses every week for a month. How much money did Carl earn?
- 6. Nell, who is the oldest, was to get only 1 cent for each verse she learned. She learned 3 verses each week for a month. How much did Nell earn?

- 7. Which of these children earned the least money? Why? Which one learned the most verses?
- 8. Jack made 2 large kites. Fred gave him 2 new lead-pencils for each kite. How many pencils did Jack get?
- 9. Fred had just paid 5 cents each for the leadpencils. How much money did the kites really cost him?
- 10. Nellie bought 3 apples at 2 cents each, and 2 lemons at 4 cents each. How much did she pay for the apples? How much for the lemons? How much for both lemons and apples?
- 11. Nita wrote 2 rows of words on her slate, and there were 5 words in each row. How many words in all did she write?
- 12. May wrote 2 rows of words of 10 words each.

 How many words did May write?
- 13. In my spool-box there are 3 rows of spools of 3 spools in each row. How many in all?
- 14. Annie has 2 such boxes of spools. How many spools has Annie?
- 15: How many legs have half-a-dozen sparrows?
- 16. My desk is 2 times 8 inches wide. How wide is it?

 $4 \times 5 =$

LESSON XXI.

OBJECT AND SLATE WORK.

Work out with counters, copy and complete.

Copy, complete, and afterward prove with counters.

$$2 \times 2 =$$
 $2 \times 9 =$ $2 \times 10 =$ $3 \times 3 =$ $5 \times 3 =$ $5 \times 4 =$ $7 \times 2 =$ $4 \times 4 =$ $4 \times 2 =$ $3 \times 6 =$ $2 \times 8 =$ $2 \times 6 =$

ORIGINAL PROBLEMS.

. Make as many problems as you can from these.

$$? \times ? = 12$$
 $? \times ? = 18$ $? \times ? = 16$

RAPID ORAL CALCULATIONS.

Read and give the answers at once.

3 twos are 5 fours are	7 twos are 3 fives are	4 fours are 5 twos are
2 fives are	6 twos are	2 eights are
$2 \times 4 =$	$10 \times 2 =$	2× 9 =
$6 \times 3 =$	$4 \times 3 =$	$2 \times 2 =$

 $3 \times 5 =$

DIVISION-CASE I.

Finding out how many parts there are in a number when we know what one of the parts is.

LEARNING THE SIGN ÷.

- 1. If we have 10 marbles, to how many boys can we give 2 marbles each?
 - How many marbles are we to give to each boy? Then 2 is the number in each part, and what we want to find out is, how many of these parts there are.
 - Let us find how many 2's in 10 marbles, thus:

- How many parts do we find there are? Then how many boys will get 2 marbles each?

 In ten there are five 2's.
- · 2. The shorter way of writing this is: $10 \div 2$'s = 5.
 - 3. For what words is this sign \div used?
 - 4. Write these sentences with figures and signs in place of the words and dashes:
 - There are fours in eight. There are threes in nine. There are fives in ten. There are fours in twelve. There are threes in fifteen. There are fours in sixteen. There are sixes in eighteen. There are nines in eighteen. There are fives in twenty.

GONCRETE APPLICATIONS. Words to be learned by sound.

e ọuld	sēa'şon	dĭ-vīde'	pēo'ple
would	va- e ā $'tion$	$oldsymbol{\epsilon}$ ăn' $ ext{dles}$	€är'-färe

- 1. If Jane pays 18 cents for half-a-dozen candles, how much is that for each candle?
- 2. May earned 18 cents making lamp-lighters, at 3 cents a dozen. How many dozen did she make?
- 3. If May gets 3 cents for a dozen lamp-lighters, how many does she make for one cent?
- 4. Carl bought 16 marbles for 4 cents. How many marbles could he get for 1 cent?
- 5. If you pay 16 cents for 8 lemons, how much would one lemon cost you?
- 6. Our school had 14 days vacation at Christmas. How many weeks was that?
- 7. May had a dime and a five-cent piece. She changed them for three-cent pieces. How many three-cent pieces did she get?
- 8. How many two-cent stamps can you buy for two dimes?
- 9. I have a dozen and a half nuts to divide among 9 boys. How many can I give to each boy?

- 10. The 12 months of the year make the four seasons. How many months to each season? What are the names of the seasons?
- 11. If a paper doll costs 2 cents, how many can you buy for 8 cents? How many for 14 cents? For 10 cents? For 4 cents? For 6¢? (¢ stands for cents.)
- 12. May has a spool-box with nine spools in it.

 There are three spools in each row. How many rows are there?
- 13. I spent 20 \(\text{for car-fare to-day.} \) How many times did I ride? (Fare, 5 cents.)
- 14. How many times could I ride for 10 ¢?
- 15. How many times could I ride for 15 ¢?
- 16. How many times could you and I ride together for 20¢?
- 17. Where Carl lives, the fare is 6¢ for grown people, children half-fare. How many times could Carl and his father ride for 18¢?
- 18. A yard is equal to 3 feet. Then how many yards long is our hall, which is 15 feet long?
- 19. Carl's fish-line is 12 feet long, how many yards is that?
- 20. The door is 8 feet high. How many yards high, and how many feet over?
- 21. How many yards are equal to 18 feet?

LESSON XXIV.

OBJECT AND SLATE WORK.

Work out with counters, copy and complete.

$$20 \div 4$$
's = ? ? $\div 6$'s = 3 $15 \div ? = 3$
 $16 \div ? = 8$ $10 \div 2$'s = ? ? $\div 3$'s = 4

Copy, complete, and afterward prove with counters.

There are

ORIGINAL PROBLEMS.

Make as many problems as you can from these.

$$18 \div ? = ?$$
 $20 \div ? = ?$ $8 \div ? = ?$ $16 \div ? = ?$ $12 \div ? = ?$ $14 \div ? = ?$

RAPID ORAL CALCULATIONS.

Read and answer at once.

How many		
3's in 6	3's in 9	4's in 8
5's in 20	2's in 4	3's in 18
7's in 14	4's in 16	5's in 15

$$12 \div 6$$
's = $20 \div 10$'s = $12 \div 4$'s = $14 \div 2$'s = $10 \div 5$'s = $15 \div 3$'s = $6 \div 2$'s =

DIVISION-CASE II.

Finding out what is one of the equal parts of a number when we know how many parts there are.

LEARNING THE FRACTIONAL EXPRESSION.

- 1. Little Annie has 15 nuts. She says she will share them with us—you, herself, and me—if we will find out what her share will be. There are three of us, so the nuts must be divided into three equal parts.
- 2. Now we know how many parts there are, and what we want to find out is what is one of the parts.
- 3. There are five 3's in 15, and if we take

 1 from each 3 we shall have 1/3

 of all of them. Then Annie's share will be how many nuts?

One third of fifteen is five.

- 4. $\frac{1}{8}$ of 15 is 5 is a shorter way of writing this:
- 5. For what words is 1/3 used?
- 6. Work out these examples with your counters.
 Write them in the shorter way:

What is one half (1/2) of six?

What is one third (1/3) of nine?

What is one fourth (1/4) of sixteen?

What is one sixth (1/6) of eighteen?

What is one tenth (1/10) of twenty?

UNEQUAL DIVISIONS.

- 1. What is one third of ten? I have ten sticks of candy to give to three boys. How must I divide it so that they will have equal shares?
- 2. By giving one stick at a time to each boy, I find that when I have given 3 sticks to each boy I have 1 stick left. This may be pictured thus: $\coprod_R \coprod_R \coprod_R \coprod_R \coprod_R$.
- 3. Now, to divide this one stick equally among them, I must break it into three equal parts—thirds—and give one third to each boy.
- 5. What would be two of the boys' shares put together? $\frac{2}{3}$ of 10 = ?
- 6. In the same way, work out with counters, and express in figures and signs, how many are $7 \div 3$, also $^{1}/_{3}$ of 7 and $^{2}/_{3}$ of 7; $13 \div 3$, also $^{1}/_{3}$ of 13; $16 \div 3$, also $^{1}/_{3}$ of 16 and $^{2}/_{3}$ of 16; $19 \div 3$ and $^{1}/_{3}$ of 19.

- 7. What is one half of nine? Take nine counters, and find first the largest number in nine that can be divided into two equal numbers. What have you left over? Now divide this one counter equally between the two groups. Then nine divided into two equal parts may be pictured thus: $||||| \cdot 9 \div 2 = 4^{1}/_{2}.$
- 8. And one of these equal parts, or 1/2 of 9 = ?
- 9. In the same way, work out with counters, and express in figures and signs, how many are $5 \div 2$ and $\frac{1}{2}$ of 5; $11 \div 2$ and $\frac{1}{2}$ of 11; $15 \div 2$ and $\frac{1}{2}$ of 15; $19 \div 2$ and $\frac{1}{2}$ of 19.
- 10. What is one fourth of thirteen? Take thirteen counters, and find first the largest number in thirteen that can be divided into four equal numbers, then divide the one counter left over equally among the four groups. Then thirteen sticks divided into four equal parts may be pictured thus: $111, 111, 111, 111, 13 \div 4 = 3^{1}/4.$
- 11. And one of these parts, or $\frac{1}{4}$ of 13 = ?
- 12. In the same way, find with counters and express in figures and signs how many are $9 \div 4$ and $\frac{1}{4}$ of 9; $17 \div 4$ and $\frac{1}{4}$ of 17; also, $\frac{1}{4}$ of 5; $\frac{1}{5}$ of 11, of 16; $\frac{1}{6}$ of 13, of 19.

LESSON XXVII.

CONCRETE APPLICATIONS. Words to be learned by sound.

grănd'moth-er blăck'board p**ĕcks** lămp'līght-erş blăck'bĕr-rieş quärts

- 1. May has 12 cherries, but Carl has only half as many. How many cherries has he?
- 2. Nita has only one third as many cherries as May. How many has Nita?
- 3. Will has ¹/₄ as many as May. How many cherries has Will?
- 4. Bertha has only ½ as many cherries as May. How many has Bertha?
- 5. There were 15 blackberries on a bush. Tom took 1/5 and Fred took 1/3 of them. How many did each boy take?
- 6. I had a plate of apples. I gave 1/4 of them to May and 1/4 to Nita. How many 4ths had I left?
- 7. I had 8 apples left. How many had I at first?
- 8. May made 17 lamp-lighters. She used one and gave $^{1}/_{2}$ of the rest to Jane. How many did she give to Jane?
- 9. Will wrote 6 words on the blackboard, which was 1/3 of the lesson. How many words in all the lesson?

- 10. I had 20 \(\text{f}. \) I spent \(\frac{1}{2} \) of it for car-fare, and gave \(\frac{1}{5} \) of the rest to May. What did she get?
- 11. I had 16 dimes. I gave 1/8 of them to May. How many dimes had I left?
- 12. I spent ½ of the dimes I had left for a sled. How much did my sled cost?
- 13. Carl has \$18. His father gave him ¹/₈ of it, and his mother gave him ¹/₉ of it. How many dollars did each give him? (The sign \$ stands for dollars.)
- 14. His grandmother gave him ¹/₂ of it, and Carl earned the rest. How much did he earn?
- 15. There are 8 quarts in 1 peck of peas. How many quarts in a half-peck?
- 16. How many quarts in a peck and a half?
- 17. Carl sold 6 quarts of nuts last fall. What part of a peck was that?
- 18. There are 12 inches in a foot. My reader is $\frac{1}{2}$ of a foot long. How many inches long?
- 19. My copy-book is 3/4 of a foot wide. How many inches wide?
- 20. Carl's tool-box is $1^{1}/_{4}$ of a foot long. How many inches long is it?
- 21. What part of a foot are 9 inches? 6 inches?
- 22. My room is 5 yards long. How many feet?

OBJECT AND SLATE WORK.

Work out with counters, copy and complete.

$$1/4$$
 of $20 = ?$? of $15 = 5$ $1/4$ of $12 = ?$? of $12 = 6$ $1/4$ of $? = 31/4$? of $16 = 4$ $1/8$ of $? = 2$ $1/9$ of $18 = ?$ $1/3$ of $? = 3$ $1/2$ of $17 = ?$ $1/2$ of $? = 7$ $1/5$ of $16 = ?$

$$4^{1}/_{2}$$
 is $^{1}/_{2}$ of ? $5^{1}/_{3}$ is $^{1}/_{3}$ of ? $2^{1}/_{6}$ is $^{1}/_{6}$ of ?

Copy, complete, and prove with counters.

$$^{1}/_{2}$$
 of $11 = ^{1}/_{7}$ of $14 = ^{1}/_{5}$ of $10 = ^{1}/_{3}$ of $18 = ^{1}/_{6}$ of $12 = ^{1}/_{8}$ of $17 = ^{1}/_{6}$ of $13 = ^{1}/_{3}$ of $19 = ^{1}/_{3}$

ORIGINAL PROBLEMS.

Make as many different problems as you can of these.

RAPID ORAL CALCULATIONS.

Read and answer at once.

$$^{1}/_{5}$$
 of $20 = ^{1}/_{8}$ of $12 = ^{1}/_{8}$ of $9 = ^{1}/_{5}$ of $15 = ^{1}/_{9}$ of $19 = ^{1}/_{5}$ of $11 = ^{1}/_{2}$ of $18 = ^{1}/_{4}$ of $12 = ^{1}/_{4}$

$$3 \text{ is } \frac{1}{8} \text{ of}$$
 $8 \text{ is } \frac{1}{2} \text{ of}$ $1 \text{ is } \frac{1}{7} \text{ of}$ $3 \text{ is } \frac{1}{4} \text{ of}$ $5 \text{ is } \frac{1}{4} \text{ of}$ $4 \text{ is } \frac{1}{3} \text{ of}$ $9 \text{ is } \frac{1}{2} \text{ of}$ $10 \text{ is } \frac{1}{2} \text{ of}$

REVIEW.

Analysis and synthesis of each number from 10 to 20.

11

$$9+2=?$$
 $8+?=11$ $11-3=?$ $3+?=11$ $?+5=11$ $6+5=?$ $?-2=9$ $11-4=?$

How many more are 11 than 8? Than 2? 5? 7?
How many less than 11 are 4? Are 3? 9? 6?
How many must you add to 7 to make 11? To
6? To 4? To 2?

Tie your counters together by 2's, and see if you can make 11 with 2's. Try the same thing with 3's; with 4's; with 5's.

Take 11 counters and see if you can divide them into groups of 2's; of 3's; of 7's; of 8's.

$$11 \div 5 = ?$$
 $11 \div 2 = ?$ $11 \div ? = 5^{1}/_{2}$ $11 \div ? = 2^{1}/_{5}$

12

What is the difference between 12 and 8? Between 12 and 7? 6 and 12? 9 and 12?
What must you add to 10 to make 12? To 8?
To 7? To 3?

$$13-3=$$
? $9+?=13$ $8+5=$? $13\div$? $=6^{1}/_{6}$ $6+?=13$ $13\div$? $=4^{1}/_{3}$? $-7=6$? $+7=13$ $13\div$ 4=? $13-2=$? $13\div$ 6=? ? $-10=3$ $5+?=13$ $4+?-13$? $+10=13$ $2+$? $=13$

How many less than 13 are 11? Are 12? 9? 6? How many more are 13 than 4? Than 8? 12? 7? Tie your counters together by 2's. Try to make 13 with 2's. Try the same thing with 4's; with 5's; 6's; 8's; 7's.

$$11+?=14$$
 $\frac{1}{7}$ of $?=2$ $?-6=8$ $9+5=?$ $7\times 2=?$ $?-5=9$ $12+2=?$? of $14=2$ $14-?=11$ $10+?=14$ $14\div 2=?$ $14-8=?$ $14\div ?=7$ $2\times ?=14$ $11+?=14$ $7+?=14$ $?+9=14$ $14-?=10$ $14-9=?$ $14-?=11$ $14-2=?$ $7+7=?$ $10+?=14$ $6+8=?$

7+?=15 15-6=? 11+4=? 15-?=9
$$\frac{1}{3}$$
 of 15=? 4+?=15 3×?=15 9+6=? 1+14=? 15÷?=3 ?-13=2 5×3=? 15-?=11 $\frac{1}{2}$ of 15=? $\frac{1}{7}$ of 15=? 3+?=15 9+?=15 ?-8=7 15÷3=? $\frac{1}{5}$ of 15=? 15-3=? 12+3=? 10+?=15 15-?=11 13+?=15 15-?=13 8+7=? 13+2=?

$$8+8=?$$
 $16-9=?$ $16\div 8=?$ $7+?=16$
 $16-?=11$ $?+11=16$ $4+?=16$ $16-14=?$
 $4\times 4=?$ $16\div ?=8$ ${}^{1}/{}_{3}$ of $16=?$ $?$ of $16=8$
 $?+4=16$ $14+2=?$ $16-?=9$ $16-?=12$
 $16-?=14$ $?$ of $16=5^{1}/{}_{8}$ $2\times 8=?$ $16\div 4=?$
 ${}^{1}/{}_{4}$ of $16=?$ $8\times ?=16$ $?-8=8$ $11+?=16$
 $3+?=16$ $16-13=?$ ${}^{1}/{}_{8}$ of $?=2$ $16-?=5$

Tie your counters together by 2's, and try to make 17 with 2's. Try the same thing with 3's; with 4's; with 5's.

$$10+?=18$$
 $18-6=?$ $16+?=18$ $18-?=11$ $18-9=?$ $13+?=18$ $18\div 6=?$ $6\times?=18$ $18\div?=6$ $9\times 2=?$ $18-?=14$ $14+4=?$ $11+?=18$ $4+?=18$ $3\times?=18$ $1/6$ of $?=3$ $1/8$ of $18=?$ $18\div 4=?$ $12+?=18$ $18\div 2=9$ $2\times?=18$ $1/9$ of $18=?$ $1/2$ of $18=?$ $9+9=?$

19

20

APPLICATIONS IN UNEQUAL DIVISIONS.

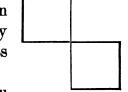
- 1. Carl has a square garden. It is 17 feet around the 4 sides of it. How long is one side?
- 2. I have 13 cakes to divide among 6 children. How much cake will each one get? $13 \div 6 = ?$
- 3. Will, Carl, and Fred have 19 yards of kitestring. How much of it belongs to each?
- 4. In 7 hours Ned walked 15 miles. How far did he walk in 1 hour? $\frac{1}{7}$ of 15 = ?

FRACTIONS.

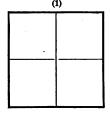
•	Words to be lear	ned by sound.		
squâre	ex-prĕss'	ĭt-sĕlf′	,	ēi′ther
	square on you exactly one inc	•	aking a	all the
2. If you	make the side it be a square?	s longer t		
3. Draw a your figur much the l	line across the new square, thus, and also by we have for the square ine, and how mu	middle of ad tell by ords how is above ch of the		
4. Is either 5. Draw a acromide	re is below the r half a square another square st the middle, the left is this left.	by itself? e inch. hen draw square.	Draw	a line
squa by fi is th	how many part re divided? E gures, thus: 4/4 e shape of these figures what	xpress it . What e parts?	1	
•	vhole square ea	-	quare is	١.

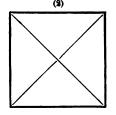
7. In one half of the square how many fourths are there? Then $\frac{1}{2} = \frac{2}{4}$, and $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$.

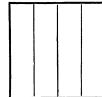
- 8. Erase one small square. How many fourths were there in all? How many did you erase? How many are left? Express all this by figures, thus: 4/4-1/4=3/4.
- 9. How many fourths have been erased here? How many fourths are left? Express by figures. $\frac{4}{4} \frac{2}{4} = ?$



- 10. How many fourths would you have to add to these to have a whole square? $\frac{2}{4} + ? = \frac{4}{4}$.
- 11. Draw another square and divide it into fourths. Erase 3 fourths. $\frac{4}{4} \frac{8}{4} = ?$





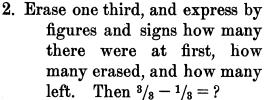


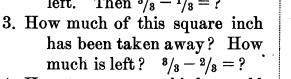
- 12. Are the parts of these squares of the same shape? Of the same size? If they were plates of gold, would $\frac{1}{4}$ of the first cost as much as $\frac{1}{4}$ of the second or of the third?
- 13. Copy these three squares, and make all the problems you can by erasing parts, telling by figures and signs all that you do.

THIRDS, SIXTHS, AND NINTHS.

1.	Draw a square inch	on your sla	ate, divide it
	into three equal	parts, and	express the
	mumban and the	i-a af Aba	_

number and the size of the parts by figures, thus: 3/3.





4. How many more thirds would make a whole inch? Then $\frac{1}{3} + ? = \frac{3}{3}$.

5. Draw another square and divide it into thirds.

Then draw a line across the middle of each third. Into how many parts does this divide each third?

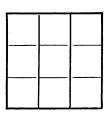
6. Into how many parts does it divide the whole square?

Express it in figures, thus: 6/6.

7. In one third, how many sixths are there? Then 1/3 = 2/6, and 1/2 of 1/3 = 1/6.

8. How many times could you erase $\frac{2}{6}$ at a time? $\frac{3}{6}$ at a time?

- 9. Erase $\frac{1}{6}$, and tell what is left. $\frac{6}{6} \frac{1}{6} = ?$
- 10. Erase $\frac{1}{6}$ at a time and tell: $\frac{6}{6} \frac{2}{6} = ?$ $\frac{6}{6} \frac{3}{6} = ?$ $\frac{6}{6} \frac{4}{6} = ?$ $\frac{6}{6} \frac{5}{6} = ?$
- 11. How many 6ths are $\frac{2}{6}$ and $\frac{1}{6}$? $\frac{2}{6} + \frac{4}{6} = ?$ $\frac{5}{6} + \frac{1}{6} = ?$ $\frac{3}{6} + \frac{1}{6} = ?$ $\frac{3}{6} + \frac{2}{6} = ?$
- 12. Draw another square inch and divide it into thirds. Divide each third into three equal parts, as in this square. Express in figures the number and size of the parts, thus: 9/9.



- 13. How many 9ths in each third? Then $\frac{1}{3} = \frac{8}{9}$, and $\frac{1}{3}$ of $\frac{1}{3} = ?$
- 14. How many times could you erase 3/9?
- 15. How many 9ths are $\frac{6}{9}$ and $\frac{3}{9}$? $\frac{5}{9} + \frac{4}{9} = ?$ $\frac{2}{9} + \frac{7}{9} = ?$ $\frac{4}{9} + \frac{8}{9} = ?$ $\frac{8}{9} + \frac{8}{9} = ?$ $\frac{5}{9} + \frac{2}{9} = ?$ $\frac{4}{9} + \frac{4}{9} = ?$
- 16. Erase $\frac{3}{9}$ and tell how many 9ths are $\frac{9}{9}$ less $\frac{3}{9}$. $\frac{9}{9} \frac{5}{9} = ?$ $\frac{9}{9} \frac{6}{9} = ?$ $\frac{9}{9} \frac{7}{9} = ?$ $\frac{9}{9} \frac{1}{9} = ?$ $\frac{9}{9} \frac{4}{9} = ?$ $\frac{9}{9} \frac{8}{9} = ?$
- 17. Draw three squares; divide one into thirds, one into sixths, and the other into ninths, and tell:

How many 6ths are equal to $\frac{2}{3}$? To $\frac{1}{2}$? $\frac{3}{6}$? How many 9ths are equal to $\frac{2}{6}$? To $\frac{4}{6}$? $\frac{2}{8}$? How many 3ds are equal to $\frac{3}{9}$? To $\frac{2}{6}$?

PART IV.

Operations in Numbers above Twenty.

LESSON I.

NOTATION AND NUMERATION IN TENS TO ONE HUNDRED.

1. Arrange your counters in groups, like these:

How many tens are there here?

How many ones?

Units is another word to use instead of ones; it means the same thing.

Two tens and five units.

- 2. Add another ten to your counters. Write how many tens and how many units.
- 3. Add another ten. Write, as before, how many tens you have now, and how many units.
- 4. Beside your sentences write the figures which

express the same thing, thus:	t. u.
Two tens, five units.	2.5
Three tens, five units.	35
Four tens, five units.	45

5. Add five more tens, one ten at a time. Write:

Five tens, five units.	55
Six tens, five units.	65
Seven tens, five units.	7 5
Eight tens, five units.	85
Nine tens, five units.	95

6. Take away the five single counters. What have you left? How many tens? How many units?

Nine tens and no units, 90.

- 7. Copy the sentences in 4 and 5, putting the word no in place of the word five.
- 8. Beside the sentences write the figures 20 which express the same thing, thus: 30
- 9. In these numbers, tell which figures etc. stand for tens, and how many tens.
 - 25, 52, 63, 27, 78, 45, 38, 82, 19, 85.
- 10. Read these numbers; tell first how many tens and units, and then give the name:
 - 20, 17, 36, 42, 55, 40, 39, 28, 67, 50, 73, 65, 98, 19, 87, 79, 48, 85, 92, 84.
- 11. Read quickly: 12, 22, 33, 44, 55, 66, 77, 88, 99, 23, 34, 45, 56, 67, 78, 89, 91.
- 12. Read the numbers in 10 and 11, glancing at two numbers at a time, and looking off the book when saying them.
- 13. Write in columns the names and the figures for all numbers from fifteen to twenty-five; from twenty-six to thirty-seven; from forty-nine to thirty-eight; from fifty to seventy-five; from ninety-nine to seventy-six.

ADDITION, IN SUMS BELOW ONE HUNDRED.

- 2. Place a bundle of counters beside the five.

 What number do you make by putting ten to five? How many counters in all? 15+4=?
- 3. Add another ten to your left-hand group, and tell what number you have made. 25+4=?
- 4. Add another ten. 35+4=? Add another ten. 45+4=? Add another ten. 55+4=?
- 5. Add another ten. 65+4=? Add another ten. 75+4=? Add another ten. 85+4=? Add another ten. 95+4=?
- 6. Place your counters in groups of 2 and 6.

 Make all the examples you can of 2+6,
 by adding 9 tens, 1 ten at a
 time, to the 2 ones. As you 2+6=8make these additions with 12+6=18your counters, write them, 22+6=28one under another, in the etc., to
 form of a table, thus: 92+6=98

- 7. Place your counters again in groups of 2 and 6. Add 9 tens, 1 ten at a time, to the 6 ones (instead of to the 2+6=8 2 ones). Write out these addi-2+16=18 tions as you make them, in 2+26=28 the form of a table, thus, and etc., to compare with the table for 2+96 "2+6, 12+6, 22+6," etc.
- 8. In the same way, make and write two tables of additions for 4+3; also, two tables for 5+2; for 6+3; for 3+5; for 7+2; for 6+4.
- 9. Arrange your counters in groups of 5 and 7. 5+7=?
- 10. Now, make a table of additions by adding 8 tens, 1 ten at a time, to the 5. Write the table as you make it.
 - Do not "make up the ten" formed by adding 5 and 7, for you would have to untie the 10 and arrange again in groups of 7 and 5 for each new addition.
- 11. In the same way, make and write a table of additions for 8+7, and one for 7+8, and compare the two tables.
- 12. In the same way, make and write tables for 9+6 and 6+9; for 8+4 and 4+8; for 5+6 and 6+5; for 9+7 and 7+9; for 8+6 and 6+8.

CONCRETE APPLICATIONS.

- 1. Carl has 67 cents in the bank and 8 cents in his purse. How much in all has he?
- 2. I found 46 carnations this morning, and 9 this afternoon. How many in all?
- 3. I spent all the month of August and 7 days of September in the country. How many days in all?
- 4. Carl paid 46 cents for a fishing-pole and 8 cents for fish-hooks. How much for both?
- 5. Mary is 17 years old and Anna is seven years older than Mary. How old is Anna?
- 6. How many inches long is a board that is a foot and 9 inches long?
- 7. There are 60 minutes in an hour. How many minutes in an hour and a half?
- 8. May had a quarter of a dollar, and Will gave her 9 cents more. How much has she now?
- 9. How many hours in one day? How many in a day and a half?
- 10. Will is 2 feet and 9 inches taller than little Fred. How many inches is that.
- 11. Fred is 2 feet and 3 inches tall. How tall, then, is Will?
- 12. How many cents in 3 dimes? In 5 dimes?
- 13. How many dimes in \$3? In \$5?

OBJECT AND SLATE WORK.

Work out with counters, copy and complete.

$$7+36 = 9+18 = 19+5 = 7+14 = 37+6 = 29+8 = 5+59 = 74+7 = 67+6 = 8+79 = 25+9 = 4+47 = 76+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 44+7 = 68+9 = 5+39 = 68+9 = 5+39 = 68+9 = 5+39 = 68+9 = 5+39 = 68+9 =$$

Copy, complete, and prove with counters.

$$1+2+3+4+5+6+7+8+9+10 = 17+6+3+4+2+1+2+3+2+5 = 10+9+8+7+6+5+4+3+2+1 = 3+37 = 18+3 = 76+5 = 6+19 = 47+3 = 83+8 = 5+66 = 96+9 = 53+7 = 3+88 = 55+6 = 69+6 = 69+6$$

RAPID ORAL CALCULATIONS.

Read, stating sums only, thus: "8, 16, 32," etc.

Begin with 1 and add 4's to 41. Begin with 3 and add 4's to 43. Begin with 2 and add 5's to 52. Begin with 2 and add 7's to 72.

SUBTRACTION, IN NUMBERS BELOW ONE HUNDRED.

- 1. Place 9 counters on your desk; take away 4 of them. What have you left?

 9 less 4 are 5.
- 2. Work out these examples with your counters, copy them in the form of a table, and complete: 19-4; 29-4; 39-4; 49-4; 59-4; 69-4; 79-4; 89-4; 99-4.
- 3. In the same way, work out and write a table of subtractions of 8-6, etc., to 98-6.
- 4. In the same way, work out and write a table of subtractions of 7-3, etc., to 97-3.
- 5. In the same way, work out and write a table of subtractions of 6-5, etc., to 96-5; also, one of 5-3, etc., to 95-3; also, of 4-2, etc., to 94-2; of 3-3, etc., to 93-3.
- 6. How many more are 9 counters than 4 counters? Place 9 counters on your desk; under these place 4 counters.
- 7. Now, to find the difference between these two numbers, we will take as many counters from the greater number (9) as there are counters in the smaller number (4), and what is left will be the difference between them.

 Then 9 are 5 more than 4. 9-4=5.

- 8. Notice that we express these two questions, "9 less 4 are how many?" and "9 is how many more than 4?" by figures and signs, in exactly the same form, thus: "9-4=?"
- 9. In the same way find how many more are
 - 9 than 6 7 than 5 6 than 2 3 than 1
 - 19 than 6 27 than 5 16 than 2 13 than 1
 - 39 than 6 57 than 5 56 than 2 23 than 1
 - 69 than 6 77 than 5 86 than 2 73 than 1

CONCRETE APPLICATIONS.

- 1. How much older is Anna, who is 19 years old, than Frank, who is only 6 years old?
- 2. John had 48 marbles, but he has lost half a dozen. How many has he left?
- 3. There were 97 trees in the park. The wind blew down 6 of them. How many are left?
- 4. I have 32 buttons on a string, but Anna has 38. How many less have I than Anna?
- 5. Our hall is 40 inches wide. Will yard-wide carpet cover it from side to side? How much wider is the hall than the carpet?
- 6. Nellie found 4 dozen eggs in the barn last week, and only 45 this week. How many less this week than last?

OBJECT AND SLATE WORK.

Work out with counters, copy, and complete these examples, writing the sign — in place of the word "less."

How	many	are						
17	less 4	16 less	3 1	8 less	5	19	less	7
27	less 4	26 less	3 2	28 less	5	99	less	7
57	less 4	36 less	3 5	8 less	5	29	less	7
18	less 4	14 less	3	5 less	4	17	less	6
88	less 4	24 less	3 2	25 less	4	27	less	6
8	less 4	34 less	3 8	35 less	4	77	less	6

Copy and complete, writing the sign — in place of the words "than" and "and."

How many n	nore are				
13 than 3	18 than 6	59 than 5	26 than 4		
33 than 3	28 than 6	69 than 5	46 than 4		
$73 ext{ than } 3$	98 than 6	79 than 5	86 than 4		
What is the difference between					
19 and 8	45 and 2	38 and 7	69 and 3		
39 and 8	85 and 2	18 and 7	79 and 3		
59 and 8	95 and 2	58 and 7	89 and 3		

RAPID ORAL CALCULATIONS.

$$27-7=$$
 $39-5=$ $26-5=$ $79-8=$ $45-4=$ $56-4=$ $63-3=$ $98-7=$ $37-5=$ $78-7=$ $95-1=$ $67-6=$

- Arranging the counters are how many?

 Arranging the counters thus,
 we see at once that we can not
 take 7 ones from 3 ones, so we untie one
 of the tens and arrange
 the counters thus, having
 1 ten and 13 single counters. Now, taking
 the 7 ones from the 13 ones, we find that
 we have six ones and one ten (16) left.
 The work we have done with our counters
 is expressed by figures and signs thus:
 23 7 = 16.
- 2. In the same way, find and express how many are 45-7; 37-8; 56-9; 72-4; 88-9; 24-5; 61-6; 93-8.
- 3. Make with counters and write a table of subtractions, by taking 7 counters from each of these numbers: 15, 25, 35, 45, 55, 65, 75, 85, 95.
- 4. In the same way, work out and write tables of subtractions for 17-9, etc., to 97-9; 13-5, etc.; 14-6, etc.; 12-7, etc.; 11-3, etc.; 18-9, etc.; 16-7, etc.; 12-4, etc.; 19-11, etc.; 11-4, etc.; 13-6, etc.; 14-5, etc.; 15-9, etc.; 16-8, etc.; 13-9, etc.; 14-8, etc.

- 5. How many more are 23 than 7? Place 23 counters on your desk; place 7 counters under them. To find the difference between these two numbers, we must take as many counters from the larger number (23) as there are counters in the smaller (7). Can you take 7 ones from 3 ones? What, then, must you first do? Work out the answer to this question with counters, and express it in figures and signs, thus: 23-7=16.
- 6. In the same way, find and express the difference between 24 and 9; 27 and 8; 31 and 6; 43 and 9; 52 and 5; 66 and 9; 81 and 8.

CONCRETE APPLICATIONS.

- 1. In seven years John will be 34 years old. How old is he now?
- 2. Carl's kite-tail was 27 feet long, but 8 feet of it were torn off. How much is left?
- 3. Nellie had 42 buttons on her button-string; now she has 34. How many has she lost?
- 4. A table is 3 feet wide, and is 7 inches longer than it is wide. How many inches long is it?
- 5. Walter is 8 and his brother is 7 years old.
 What is the difference in their ages?

- 6. Carl had 55 cents; he spent 9 cents for slatepencils and candy. How much had he left?
- 7. This afternoon he has spent 8 cents more, for writing-paper. How much remains now?
- 8. Harry and Ned have, together, 43 marbles; 9 of them are Ned's. How many are Harry's?
- 9. Tom had 35 pop-corn balls to sell on the trains. On the 4 o'clock train he sold 8. How many had he left?
- 10. On the 4.15 train he sold 9 more. How many had he then left? On the 4.30 train Tom sold all the rest but 9. How many did he sell?
- 11. Will's top-cord was 1 yard, 1 foot long. He broke off 9 inches. How long was it then?
- 12. Tom had a quarter of a dollar, and spent 6 cents. How much was left?
- 13. Nell had a basket of 2 dozen eggs; she let the basket tip over, and 8 eggs fell out and were broken. How many were left?
- 14. If you sleep 7 hours, how many hours of the day are you awake?
- 15. 9 of the 52 weeks of this year are past; how many are still to come?
- 16. In my garden to-day there were 33 carnations; I picked 7. How many did I leave?

OBJECT AND SLATE WORK.

Work out with counters, copy, and complete.

$$97 - 7 - 4 - 6 - 5 - 5 - 2 - 8 - 1 - 9 =$$
 $89 - 5 - 6 - 7 - 9 - 3 - 2 - 8 - 6 - 5 =$
 $58 - 5 - 6 - 7 - 9 - 3 - 2 - 8 - 6 - 5 =$
 $85 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 1 =$

Copy, complete, and prove with counters.

RAPID ORAL WORK.

Read, stating remainders only, thus: "30, 27, 24," etc.

$$30-3-3-3-3-3-3-3-3-3-3=$$
 $48-6-5-4-5-7-3-5-10=$
 $90-20-5-5-2-5-5-10-5=$
 $78-18-8-7-2-1-6-5-9-8=$

Begin with 40, and take as many 4's as you can. Begin with 38, and take 3's; 5's; 7's; 9's. Begin with 54, and take 8's; 9's; 2's; 7's; 6's. Begin with 67, and take 6's; 8's; 4's; 9's; 5's.

MULTIPLICATION.

Making and Learning the Tables.

- 1. Group your counters by threes until you have 10 threes. How many counters in all is this? Copy and complete each of these examples as you find the answer, and write in the form of a table. How many are three 3's? Five 3's? Seven 3's? Nine 3's? Ten 3's? Eight 3's? Six 3's? Four 3's? Two 3's?
- 2. Arrange your counters in 10 groups of 4 each. Find answers, copy, complete, and write as a table, how many are two 4's? Four 4's? Six 4's? Eight 4's? Ten 4's? Nine 4's? Seven 4's? Five 4's? Three 4's?
- 3. Arrange your counters in 10 groups of 5 each. Find answers, copy, and complete. How many are 3×5 ? 5×5 ? 7×5 ? 9×5 ? 2×5 ? 4×5 ? 6×5 ? 8×5 ? 10×5 ?
- 4. Arrange your counters in 10 groups of 6 each.

 How many are one 6? Two 6's? Three 6's?

 Four 6's? Five 6's? Six 6's? Seven 6's?

 Eight 6's? Nine 6's? Ten 6's?
- 5. Arrange your counters in 10 groups of 7 each. How many are 7×7 ? 9×7 ? 3×7 ? 6×7 ? 2×7 ? 4×7 ? 8×7 ? 5×7 ? 10×7 ?

- 6. Arrange your counters in 10 groups of 8 each.

 How many are two 8's? Four 8's? Eight
 8's? Three 8's? Six 8's? Nine 8's? Five
 8's? Ten 8's? Seven 8's?
- 7. Arrange your counters in 10 groups of 9 each. How many are 10×9 ? 9×9 ? 8×9 ? 7×9 ? 6×9 ? 5×9 ? 4×9 ? 3×9 ? 2×9 ?
- 8. Arrange your counters in 10 groups of 10 each. How many are 2×10 ? 3×10 ? 4×10 ? 5×10 ? 6×10 ? 7×10 ? 8×10 ? 9×10 ? 10×10 ?

DICTATION EXERCISE.

- 1. How many dots in this square of 5 dots each way? $5 \times 5 = ?$
- 2. Make a square of dots, four dots $\bullet \bullet \bullet \bullet \bullet$ each way. How many dots in all? $4 \times 4 = ?$
- 3. Make a square of 7 dots each way. $7 \times 7 = ?$
- 4. Make a square of 8 dots each way. $8 \times 8 = ?$
- 5. Make a square of 6 dots each way. $6 \times 6 = ?$
- 6. Make a square of 9 dots each way. $9 \times 9 = ?$
- 7. Make a square of 3 dots each way. $3 \times 3 = ?$
- 8. Make a square of 2 dots each way. $2 \times 2 = ?$
- 9. Make a square of 10 dots each way. $10 \times 10 = ?$
- 10. Complete, and learn this table of squares.

CONCRETE APPLICATIONS.

- 1. John earns \$3 a week. How much does he earn in 2 months? (\$ stands for dollars.)
- 2. How many shoes will it take to shoe 7 horses?
- 3. How many days are there in 8 weeks?
- 4. How many school-days in 9 weeks?
- 5. Ned has 6 quarts of strawberries. How many pint-baskets can he fill with them? (There are 2 pints in 1 quart.)
- 6. How many quart-baskets could you fill from 1 peck of plums? (There are 8 quarts in a peck.) How many from 2 pecks? 3 pecks?
- 7. How many sides have six and the next 7 examples.
- 8. How many sides have 9 triangles?
- 9. How many faces have 7 cubes?
- 10. How many sides have 8 pentagons?
- 11. Four pentagons have as many sides as how many squares?
- 12. Three pentagons have as many sides as how many triangles?
- 13. Three squares have as many sides as how many triangles?
- 14. Make 12 pentagons, 20 triangles, and 15 squares, and count the sides in each set of figures.

OBJECT AND SLATE WORK. Copy, complete, and learn.

$$1 \times 2 = 6 \times 2 = 1 \times 3 = 6 \times 3 = 1 \times 4 = 6 \times 4 = 2 \times 2 = 7 \times 2 = 2 \times 3 = 7 \times 3 = 2 \times 4 = 7 \times 4 = 3 \times 2 = 8 \times 2 = 3 \times 3 = 8 \times 3 = 3 \times 4 = 8 \times 4 = 4 \times 2 = 9 \times 2 = 4 \times 3 = 9 \times 3 = 4 \times 4 = 9 \times 4 = 5 \times 2 = 10 \times 2 = 5 \times 3 = 10 \times 3 = 5 \times 4 = 10 \times 4 = 1$$

Work out with counters, copy, complete, and learn.

$$1 \times 5 = 6 \times 5 = 1 \times 6 = 6 \times 6 = 1 \times 7 = 6 \times 7 = 2 \times 5 = 7 \times 5 = 2 \times 6 = 7 \times 6 = 2 \times 7 = 7 \times 7 = 3 \times 5 = 8 \times 5 = 3 \times 6 = 8 \times 6 = 3 \times 7 = 8 \times 7 = 4 \times 5 = 9 \times 5 = 4 \times 6 = 9 \times 6 = 4 \times 7 = 9 \times 7 = 5 \times 5 = 10 \times 5 = 5 \times 6 = 10 \times 6 = 5 \times 7 = 10 \times 7 = 1 \times 8 = 6 \times 8 = 1 \times 9 = 6 \times 9 = 1 \times 10 = 6 \times 10 = 2 \times 8 = 7 \times 8 = 2 \times 9 = 7 \times 9 = 2 \times 10 = 7 \times 10 = 3 \times 8 = 8 \times 8 = 3 \times 9 = 8 \times 9 = 3 \times 10 = 8 \times 10 = 4 \times 8 = 9 \times 8 = 4 \times 9 = 9 \times 9 = 4 \times 10 = 9 \times 10 = 5 \times 8 = 10 \times 8 = 5 \times 9 = 10 \times 9 = 5 \times 10 = 10 \times 10$$

RAPID ORAL CALCULATIONS.

9	six 6's =	$9 \times 7 =$	$7 \times 8 =$	eight 6 's =
8	nine $9's =$	$8 \times 4 =$	$6 \times 9 =$	nine $8's =$
5	eight 8's =	$5 \times 8 =$	$5 \times 7 =$	six 7's =

DIVISION-CASE I.

- 1. Arrange 24 counters on your desk; separate them into groups of six. How many 6's are there? $24 \div 6 = ?$
- 2. In the same way, work out with your counters, copy, and complete these examples:

$$27 \div 9 = 36 \div 4 = 20 \div 4 = 30 \div 5 = 42 \div 6 = 56 \div 7 = 16 \div 4 = 25 \div 5 = 36 \div 6 = 49 \div 7 = 64 \div 8 = 81 \div 9 = 48 \div 6 = 54 \div 9 = 60 \div 6 = 6$$

3. Also work out with your counters, copy, and complete these:

$$24 \div 4 = 35 \div 5 = 48 \div 8 = 70 \div 7 = 72 \div 9 = 28 \div 7 = 40 \div 5 = 54 \div 6 = 21 \div 7 = 32 \div 8 = 63 \div 7 = 45 \div 9 = 30 \div 6 = 24 \div 8 = 28 \div 4 = 2$$

CASE II.

- 4. Separate 32 counters into 8 equal groups, and find and tell how many counters in one of these groups. $\frac{1}{8}$ of 32 = ?
- 5. Work out with counters in the same way, and copy and complete these examples:

$$\frac{1}{3}$$
 of $9 = \frac{1}{6}$ of $18 = \frac{1}{4}$ of $24 = \frac{1}{9}$ of $27 = \frac{1}{4}$ of $28 = \frac{1}{9}$ of $36 = \frac{1}{7}$ of $21 = \frac{1}{6}$ of $42 = \frac{1}{8}$ of $56 = \frac{1}{7}$ of $63 = \frac{1}{6}$ of $54 = \frac{1}{9}$ of $81 = \frac{1}{4}$ of $16 = \frac{1}{9}$ of $72 = \frac{1}{5}$ of $35 = \frac{1}{8}$ of $72 = \frac{1}{5}$

DICTATION EXERCISES.

- 1. Make 36 dots on your slate, so that there will be the same number of dots each way. The square of what number is 36?
- 2. Arrange 64 dots in a square. The square of what number is 64?
- 3. Arrange 25 dots in a square. 25 is the square of what number?
- 4. Arrange 49 dots in a square. 49 is the square of what number?
- 5. Arrange 81 dots in a square. 81 is the square of what number?
- 6. Find, by trying, which of these numbers can be arranged in squares and which can not: 35, 40, 16, 22, 12, 9, 24, 48, 49, 88, 72, 64.

CONCRETE APPLICATIONS.

- 1. Our house is 27 feet wide. How many yards wide is that? (3 feet in 1 yard.)
- 2. How many hours do I sleep if I sleep 1/4 of the day? (A day is 24 hours.)
- 3. I have studied ¹/₆ of an hour. How many minutes is that? (An hour is 60 minutes.)
- 4. Carl has 36 marbles, 9 in each of his pockets. How many pockets has he?

- 5. Nita has set up 45 blocks in 5 rows. How many in each row?
- 6. If you make 72 dots in 8 rows, how many dots will there be in each row?
- 7. How many rows would there be if you make them rows of 12 each?
- 8. How many rows if in rows of 24 each?
- 9. Carl ate 8 plums, which were 1/7 of all he had. How many had he at first.
- 10. Nita is 6 years old, which is 1/5 of her mother's age. How old is her mother?
- 11. How many quart bottles can be filled from 24 pints of milk? (2 pints in 1 quart.)
- 12. How many gallon jugs would be filled from 32 quarts? (4 quarts in 1 gallon.)
- 13. John made 40 cents to-day selling pop-corn balls at 2 for 5 cents. How many did he sell?
- 14. Thomas earns ¹/₈ as much money a month as his father does, whose wages are \$48 a month. How much does Tom earn?
- 15. Aunt Sarah says that when I have earned 40 cents she will give me ¹/₄ as much. How much shall I then have?
- 16. I have 1/6 as many marbles as Tom, who has 42. How many have I?

OBJECT AND SLATE WORK. Write the complete answers.

How m	any times		•		
3 in	4 in	5	in	6 in	7 in
27	36	3	5	36	49
30	24	3	80	42	42
21	32	4	5	54	$\bf 56$
How m	any				
8's	in	9 's	in	10	0's in
24	72	27	45	2 0	30
32	56	63	73	40	60
16	64	54	81	80	90
40	4 8	36	4 8	10	100

Work out with counters, copy and complete.

$$36 \div 7 =$$
 $28 \div 6 =$ $\frac{1}{3}$ of $28 =$ $\frac{1}{6}$ of $44 =$ $41 \div 5 =$ $38 \div 4 =$ $\frac{2}{5}$ of $35 =$ $\frac{3}{8}$ of $32 =$

RAPID ORAL CALCULATIONS.

$$36 \div 9 =$$
 $32 \div 8 =$ $63 \div 9 =$ $35 \div 7 =$ $54 \div 6 =$ $49 \div 7 =$ $72 \div 8 =$ $81 \div 9 =$

$$^{1}/_{2}$$
 of $16 = ^{5}/_{6}$ of $36 = ^{1}/_{7}$ of $63 = ^{1}/_{7}$ of $56 = ^{1}/_{4}$ of $28 = ^{1}/_{4}$ of $32 = ^{3}/_{8}$ of $64 = ^{5}/_{8}$ of $40 = ^{2}/_{5}$ of $35 = ^{3}/_{5}$ of $45 = ^{1}/_{9}$ of $54 = ^{3}/_{6}$ of $42 = ^{2}/_{1}$

FRACTIONS.

- 1. Draw a square inch. Draw a line down the middle. Into how many equal parts does this divide it? Express this in figures.
- 2. Divide each half into two equal parts. How many parts in all, now? $\frac{1}{2}$ of $\frac{1}{2}$ =?
- 3. Now, divide your square as this one is divided, by drawing a line through the middle of each fourth. Into how many parts does this divide it?

 Then 1/2 of 1/4 =?



- 4. How many 8ths in $\frac{1}{4}$? $\frac{3}{4} = \text{how many 8ths}$? $\frac{1}{2} = \text{how many 8ths}$?
- 5. How many times $\frac{2}{8}$ are there in the square? Then $\frac{8}{8} \div \frac{2}{8} = 4$. $\frac{1}{4}$ of $\frac{8}{8} = ?$
- 6. How many times $\frac{4}{8}$ are there? $\frac{1}{2}$ of $\frac{8}{8} = ?$
- 7. How many more are $\frac{8}{8}$ than $\frac{3}{8}$? $\frac{6}{8}$ + ? = $\frac{8}{8}$. $\frac{4}{8}$ + ? = $\frac{8}{8}$. $\frac{3}{8}$ + $\frac{2}{8}$ = ? $\frac{2}{8}$ + ? = $\frac{7}{8}$. $\frac{5}{8}$ $\frac{1}{8}$ = ? $\frac{7}{8}$? = $\frac{4}{8}$.
- 8. How many 8ths are $\frac{8}{8}$ less $\frac{2}{8}$? $\frac{8}{8} \frac{5}{8} = ?$ $\frac{8}{8} \frac{7}{8} = ?$
- 9. How many 8ths are 2 times $\frac{2}{8}$? $3 \times \frac{2}{8} = ?$ $2 \times \frac{3}{8} = ?$
- 10. How many times $\frac{2}{8}$ in $\frac{6}{8}$? $\frac{4}{8} \div \frac{2}{8} = ?$ $\frac{1}{2}$ of $\frac{8}{8} = ?$ $\frac{1}{4}$ of $\frac{8}{8} = ?$ $\frac{1}{8}$ of $\frac{6}{8} = ?$

ADDING BY TENS.

- 1. Do you think it is any easier to add together 3 single counters and 4 single counters, than it is to add 4 bundles of ten and 3 bundles of ten? Try it and see.
- 2. How many tens are 4 tens and 5 tens? How many single counters in 4 tens? How many single counters in 5 tens? Then 40 and 50 are how many?
- 3. How many are 6 tens and 2 tens? 60+20=? 70+20=? 40+30=? 10+50=?
- 4. How many are 3 tens and 4 tens and 2 tens? 30+40+20=? 20+10+30=?
- 5. How many are 30 + 20 + 8? 50 + 40 + 9 = ? 30 + 15 + 50 = ? 40 + 15 + 40 = ?
- 6. How many are 10+20+30+20+10+7?
- 7. Place 32 counters on your desk. Under them place 26 counters, so that the 2 tens will be under the 3 tens, and the 6 singles will be under the 2 singles, thus:

 Now add these two numbers by putting the units with the units, and the tens with the tens. What is the new number that you have made? This is called *The sum* of the two numbers.

- 8. Express in figures what you had done with your counters, by writing the numbers to be added one under the other, and 32 then draw a line under them, and write the sum of the numbers below 58 the line, thus:
- 9. Work out with your counters, and express in figures, as above, these additions: 42 and 25; 55 and 14; 62 and 27; 81 and 15; 75 and 22; 33 and 44; 51 and 36; 23 and 42 and 14; 21 and 34 and 23 and 15.
- 10. Place on your desk 47 counters, and under them 38 counters, and under these 14 counters. When you add the 8 units (single sticks) to the 7 and the 4 units, what do you find? What will you do with the tous new ten thus made? (Add it in with 47 the 1 and 3 and 4 tens.) What you 38 have done with your counters is here 14 expressed in figures.
- 11. Add 1 more counter to your 9 single counters. You can now make another ten. How many tens in all have you?
- 12. There is another name for ten tens; do you know it? Write

Ten tens make one hundred.

ADDITION, NOTATION, AND NUMERATION, IN HUNDREDS.

- 1. How many are 78 and 47? Let us picture this example, with dots for the units and rings for the tens, thus:
- 2. We add first the 8 units and the 7 units, and find that we have 15 units, or 1 ten and 5 units. Under the line, in the units' place, we will put the 5 units (dots),

and add the 1 ten (ring) to the tens (rings).

This 1 ten added to the 4 and the 7 tens makes 12 tens, or 1 hundred, and 2 tens.

3. We put the 2 tens (rings) in the tens' place, and express the 1 hundred by a large ring, which we put in the hundreds' place.

	000	0 0	 	
	0	0	·	
0	0	0		

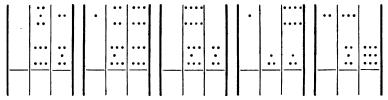
4. The work, when completed, will look like this, and it may be expressed in figures thus:

5. In this number we can tell which rings 78

| O | O | O | O | Stand for hundreds and 47
| O | O | O | O | O | O | O |

size, and second by their place, for tens are always in the second place to the left, and hundreds are always in the third place to the left.

- 6. After this we will use only the second of these two ways of showing which are units, which are tens, and which are hundreds. That is, instead of using dots and different sized rings, we will use dots only, and let their places show whether the dots stand for units, or tens, or hundreds.
- 7. So, instead of writing it thus, with dots and rings, we will express this number by dots only, thus, and it may be expressed in figures thus, 235.
- 8. Copy and complete these examples in addition, and then express them in figures:



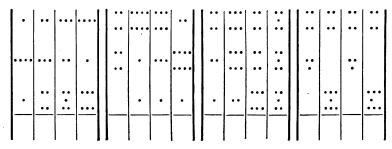
- 9. Express these numbers in words:
 - 125 728 809 367 590 982 260 541 315 619 701 400 600 780 708 611
- 10. Express in figures: one hundred, twenty-one; six hundred, forty-two; eight hundred, eighteen; seven hundred, seventy; five hundred, five; four hundred; three hundred, thirty-three; nine hundred, nineteen.

ADDITION, NOTATION, AND NUMERATION. IN THOUSANDS.

- 1. You have learned that ten units make one ten, and that ten tens make one hundred. Now, what do you think ten hundreds make?

 Ten hundreds make one thousand.
- 2. Add 5 hundreds, 7 tens, 6 units. and 7 hundreds, 4 tens, 8 units. Express these numbers by dots, thus. Now, adding the 6 and the 8 units, you find that you have 14 units, that is, 1 ten and 4 units; you put the units under the line in the units' place, and, adding this 1 ten to the 4 tens and 7 tens, you obtain 12 tens, which are equal to 1 hundred, and 2 tens. Putting the tens in the tens' place, and adding this 1 hundred to the 7 hundreds and 5 hundreds, you find that you have 13 hundreds, that is, 1 thousand and 3 hundreds. Now, put the 3 hundreds in the hundreds' place and the 1 thousand in the fourth place to the left, the thousands' place, and the answer—that is, the 576 sum of these two numbers—will be, 748 1 thousand, 3 hundreds, 2 tens, and 4 units, expressed in figures thus: 1,324

3. Copy and complete these examples in addition, and then express in figures:



- 4. Express these numbers in figures: one thousand, six hundred, fifty-four; five thousand, three hundred, ten; four thousand, twenty-seven; eight thousand, eight hundred, eighty-six; two thousand, two.
- 5. Express these numbers in words:

1,276	5,720	6,302	7,000	8,001
10,60	4,440	1,001	2,220	1,100
3,030	5,005	1,010	8,020	9,001

- 6. Ten thousands make "a ten" of thousands, just as ten ones make "a ten." This number, 10,856, is read, ten thousand, eight hundred, fifty-six.
- 7. Read these numbers: 10,000; 20,000; 30,000; 15,000; 17,000; 12,856; 14,805; 10,010; 13,927; 15,005; 16,600.

CONCRETE APPLICATIONS.

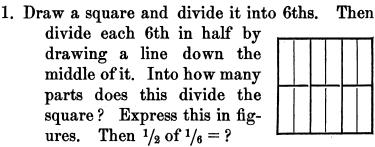
- 1. There are in my garden 35 roses, 27 carnations, and 42 sweet-peas. How many in all?
- 2. May has 125 buttons on her button-string, and Nita 108. How many have they both?
- 3. Twelve dozen of any thing are called a gross. $12 \times 12 = ?$ How many buttons in a gross and a half a gross?
- 4. In my reader there are, on the first page, 123 words, the same number on the next page, and also on the next. How many on these 3 pages? There are two ways of doing this example. See if you can find out for yourself what these ways are.
- 5. In another book there are on one page 237 words, on the next, 209, on the next, 223, on the next, 207, and on the next, 232 words. How many words on these 5 pages? Can you do this example by both addition and multiplication? Why not?
- 6. Last summer Mr. Jones raised 380 bushels of wheat, 245 bushels of oats, and 897 bushels of corn. How much grain in all?
- 7. In 1 mile there are 5,280 feet, and in a ¹/₄ of a mile 1,320 feet. How many feet in a mile and a quarter?

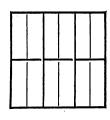
*LESSON XX.

OBJECT AND SLATE WORK.

Work out with counters, and copy and complete.

			,	.,,,	,				
45 and	36 =		9	32, 26,	13, an	100 = 0			
27 and	39 =		-	16, 26,	36, an	100 = 100			
48 and	56 =		-	12, 42,	8, an	ad 36 =			
	Write in columns and add.								
19, 9,	13, a	nd 24	13, 15	5. 16. 1	7. 18.	and 19			
	•	nd 4 12							
, ,	,		y and add		, ,				
14	19	26	15	62	8	4 8			
15	8	62	10	8	57	25			
24	27	8	36	19	20	9			
32	32	12	29	6	13	16			
			==						
327	693	427	582	1,0	069	3,686			
502	_58	$\frac{470}{}$	399	1,	672	4,864			
1,883		12,861	. 2	5,005	1	120,120			
5,972		8,542		9,699		22,903			
3,645		7,207		4,805		37,456			
808		20,101		5,062		18,379			
					-				
		RAPID ORA	IL CALCUI	ATIONS.					
30	60	20	4 0	20	47	25			
40	30	50	17	30	33	25			
20	9	18	23	3 8	15	25			

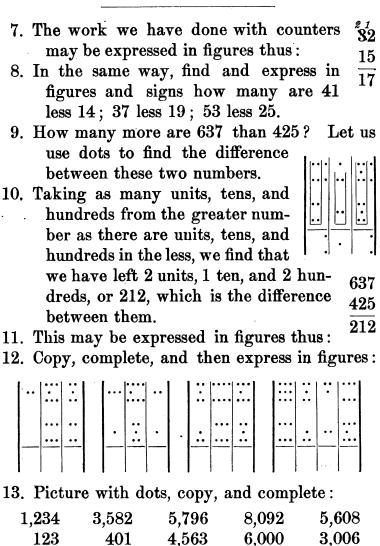




- 2. How many 12ths in half the square? Then $1/_2 = ?/_{12}$. $1/_3 = ?/_{12}$. $1/_4 = ?/_{12}$. $1/_6 = ?/_{12}$.
- 3. How many more are $\frac{12}{12}$ than $\frac{6}{12}$? $\frac{7}{12}$ +? $\frac{12}{12}$. $\frac{8}{12} + \frac{2}{12} = \frac{12}{12}$. $\frac{2}{12} + \frac{2}{12} = \frac{12}{12}$.
- 4. How many are $\frac{12}{12}$ less $\frac{5}{12}$? Less $\frac{8}{12}$?
- 5. How many are $\frac{3}{12}$ and $\frac{5}{12}$? $\frac{7}{12} + ? = \frac{9}{12}$. $4/_{12} + ? = 7/_{12}$. $9/_{12} - ? = 4/_{12}$.
- 6. How many 12ths are 3 times $\frac{4}{12}$? $2 \times \frac{6}{12} = ?$ $4 \times {}^{8}/_{12} = ? \quad 6 \times {}^{2}/_{12} = ? \quad 3 \times {}^{2}/_{12} = ?$
- 7. In $\frac{12}{12}$ how many times $\frac{2}{12}$? $\frac{4}{12}$? $\frac{6}{12}$? $\frac{8}{12}$? $\frac{1}{2}$ of $\frac{12}{12} = ?$ $\frac{1}{3}$ of $\frac{12}{12} = ?$ $\frac{1}{6}$ of $\frac{12}{12} = ?$ $\frac{1}{4}$ of $\frac{12}{12} = ?$ $\frac{1}{3}$ of $\frac{9}{12} = ?$ $\frac{1}{2}$ of $\frac{6}{12} = ?$
- 8. In how many different ways can you express this much of the square— $\frac{6}{12}$?
- 9. Answer the same question about 8/12; about $\frac{12}{12}$; $\frac{9}{12}$; $\frac{11}{12}$; $\frac{4}{12}$; $\frac{5}{12}$; $\frac{2}{12}$; $\frac{7}{12}$; $\frac{8}{12}$.

SUBTRACTION AND COMPARISON.

- 1. Is it any easier to take 4 single counters from 9 single counters than it is to take 4 tens from 9 tens? Try both, and see.
- 2. 9 tens less 4 tens =? Then 90 less 40 = ?
- 3. 8 tens less 5 tens are? 80-50=? 60-20=?
- 4. 57 less 34 are how many? Place 57 counters on your desk; from this group of 57 take out 34. What is left?
- 5. To express this in figures, you write first the number of counters you have (57); t.u. then under this write the number (34) 57 you wish to take away; then, drawing 34 a line, you write under it the figures 23 which tell how many of the 57 would remain. This is called *The Remainder*.
- 6. 32 less 15 are how many? Arranging your counters thus, we see at once that we can not take 5 ones from the 2 ones, so, untying one of the tens, we arrange the counters thus, having 2 tens and 12 ones. Then, taking from these the 1 ten and 5 ones (15), we find that we have 1 ten and 7 ones (17) left.



CONCRETE APPLICATIONS.

- 1. If there are 675 daisies in a field, and Jane picks 325, how many will be left?
- 2. Mr. Hudson raised 857 bushels of corn on his farm last summer, and has sold 569 bushels. How many bushels has he left?
- 3. He also raised 388 bushels of wheat. How much more wheat than corn did he raise?
- 4. In the flower-show there were 1,587 yellow flowers; 407 were roses. How many of other kinds of flowers?
- 5. There were 500 white flowers. How many more yellow than white flowers?
- 6. There were 859 red flowers. How many more yellow than red flowers? How many more red than white flowers?
- 7. I have walked 3,160 feet. How many more feet must I walk to make a mile? How many feet in a mile.
- 8. Will has walked 4,085 feet. How much farther has he gone than I? How many more feet must he walk to make a mile?
- 9. Frank's father made \$9,875 last year, and spent \$6,750. How much did he save?
- 10. He made \$1,790 less this year than last. What did he make this year?

*LESSON XXIV.

OBJECT AND SLATE WORK.

Work out with counters, and copy and complete these

VV 0	TK OUL WILL C	ouncers,	and copy an	ia comprete t	11686
Subtra	ctions.				
47	54	62	48	36	25
33	37	20	29	<u>16</u>	<u>17</u>
Compa	risons.				
$ar{40}$	65	79	86	4 9	82
$\frac{25}{2}$	<u>40</u>	37	<u>68</u>	<u>37</u>	<u>28</u>
	Picture v	vith dots,	and copy ar	nd complete.	
146	2 69	305	627	534	612
$\underline{25}$	<u>136</u>	103	$\underline{319}$	$\frac{324}{}$	235
		Сору а	nd complete.		
427	862		948	1,327	2,849
_116	_502	_	908	427	1,627
12,67	8 1	5,654	2 8	,028	39,267
2,56	4	5,432	8	,020	7,267
89,95	6 6	5,902	99	,090	127,342
9,05	6 6	0,092	9	,090	79,563

RAPID ORAL CALCULATIONS.

Read thus: "twenty from forty, twenty," etc.

40	60	5 8	86	333	556
20	40	50	40	120	246

MULTIPLICATION-A SHORT METHOD OF ADDITION.

- 1. Suppose we are asked to find out how many trees there would be in an orchard of 6 rows of trees with 28 trees in each row? One way to find out is by Addition; that is, to write down the number of trees in each row (28) as many times as there 28 are rows of trees (6). We would 28 then add, first, the units, thus: "8, 16, 28 24, 32, 40, 48 units—4 tens and 8 28 units"; putting down the 8 units, we 28 would then add the 4 tens to the col-**2**8 umn of tens and add thus: "4, 6, 8, $\overline{168}$ 10, 12, 14, 16 tens—1 hundred and 6 tens"-which we write in their proper places, having for our answer 168 trees.
- 2. Another and a much shorter way to find out the same thing is by Multiplication; that is, instead of writing down the 28 six times, to write it only one time, and under it to write the figure 6, to show how many times 28 trees there are; then 28 trees we multiply 28 by 6, thus, saying 6 "6 times 8 (or 6 8's) are 48—4 168 trees tens and 8 units"—we write the 8 units in their proper place; then, over

the 28 we place a small figure 4, to remind ourselves that we have these 4 tens to add to the other tens; then, after multiplying the tens, saying "6 times 2 tens are 12 tens," we add in the 4 tens, which gives us 16 tens, that is, 1 hundred, 6 tens, which we write in their proper places, and thus show that 6×28 trees = 168 trees.

- 3. Do this example both by addition and by multiplication. There are 8 rows of leaves in our hall carpet, and 24 leaves in each row; how many leaves in all?
- 4. Write 43 seven times and add; multiply 43 by 7. Add 72 five times; multiply 72 by 5.
- 5. Do these examples both ways:

38	39	27	124	672	987
7	_6	9 ·	<u>6</u>	4	2
536	881	269	9	380	1,246
5	3	8	3	4	5

6. Copy and complete these examples:

2,579	3,520	4,408	5,045
4	9	8	7
15,232	25,550	32,003	51,020
6	7	3	9

CONCRETE APPLICATIONS.

- 1. John's grandfather is 3 score and 10 years old. What is his age? (A score is 20.)
- 2. John's father is 2 score and 5 years old, and John is 5 years less than a score. What are their ages?
- 3. In a pound of sugar, or of flour, there are 16 ounces. How many ounces in 8 pounds?
- 4. There are 100 pounds in a hundred-weight. How many hundred-weight and how many pounds over in a barrel of flour (which weighs 196 pounds)?
- 5. Our horse is 15 hands high. How many inches is that (the hand-measure is 4 inches)?
- 6. There are 2 pints in a quart, 8 quarts in a peck, and 4 pecks in a bushel. How many pints in 1 bushel?
- 7. How many pint boxes could be filled from 2 bushels of strawberries?
- 8. There are 365¹/₄ days in a year. How many days in 4 years? In 6 years? In 8 years?
- 9. If there are 4 gills in a pint of vinegar, and 2 pints in a quart, and 4 quarts in a gallon, how many gills in a gallon?
- 10. How many gill bottles could be filled from 3 gallons of essence of lemon?

DIVISION.

1. Divide 6 units by 3; divide 6 tens by 3; divide 6 hundreds by 3. Thus:

$$6 \div 3 = 2$$
 $60 \div 3 = 20$ $600 \div 3 = 200$

- 2. Here is another form of expressing this same thing. It is a better form to use in dividing large numbers. $\frac{3)6}{2}, \frac{3)60}{20}, \frac{3)600}{200}.$
- 3. Copy and complete $3\underline{)9}$, $3\underline{)90}$, $3\underline{)900}$; $2\underline{)8}$, $2\underline{)80}$, $2\underline{)800}$; $4\underline{)16}$, $4\underline{)160}$, $4\underline{)1,600}$.
- 4. 888 divided by 2 equals what?

888 is equal to

8 hundreds, 2)800

8 tens, 80

8 units, 8

800 divided by 2 = 400

80 divided by 2 = 40

8 divided by 2 = 4

888 divided by 2 = 444

5. Here is a shorter way of showing the same thing:

 $\frac{2)888}{444}$

- 6. Do each of these examples in both ways: 999 $\div 3$; $666 \div 2$; $444 \div 2$; $888 \div 4$; $555 \div 5$; $848 \div 4$; $426 \div 2$; $936 \div 3$; $248 \div 2$.
- 7. Do these examples in the shorter way only: 333 \div 3; 888 \div 8; 666 \div 3; 777 \div 7; 444 \div 1; 224 \div 2; 363 \div 3; 624 \div 2; 663 \div 3; 882 \div 2.

CONCRETE APPLICATIONS.

- 1. Carl has been writing for 80 minutes, half the time on his slate and half in his copy-book. How many minutes has he been writing in his copy-book?
- 2. There are 600 minutes in ten hours. How many minutes in ¹/₈ that time?
- 3. A ranchman has 888 sheep divided into 8 flocks. How many sheep in each flock.
- 4. In a tulip-garden of 7 beds there are 777 plants, an equal number in each bed. How many is that?
- 5. In a row of ten houses there are 90 windows. How many windows to each house?
- 6. In 666 lead-pencils how many packages of a half dozen?
- 7. In 464 shoes how many pairs?
- 8. How many horses can be shod all round with 848 shoes?
- 9. I have made a certain number of triangles, and find that there are in all 393 sides. How many triangles have I made?
- 10. I have 550 \(\epsilon \) in 5-cent pieces. How many 5 \(\epsilon \) pieces?
- 11. There are 488 carriage-wheels in a shop. How many carriages will they supply.

1. What is $\frac{1}{2}$ of 90? 9 bundles of ten divided in-
to 2 equal groups gives 4 tens in
gives 4 tens in TTT TTT T
each group and 1 ten over. Untying the
ten left over, we divide it in half and find
that there are five sticks in each half,
which, added to the tens, gives 45 sticks in each group. Then $\frac{1}{2}$ of $\frac{90}{45}$
in each group. Then $\frac{1}{2}$ of $90 = 45$.

2. Suppose we wanted to find $\frac{1}{2}$ of 94. One way would be to find, first, the half of 90, just as we did before, and then find the half of the 4 and add it to the half of the 90, thus:

2)90

4

45

1/2 of 90 gives $\frac{4}{45}$ 1/2 of $\frac{4}{94}$ gives $\frac{2}{47}$

3. But a shorter way would be to find the half of the 9 tens, and, then adding the sticks of the 1 bundle of tens left over to the 4 sticks, find at once the half of all the single sticks, thus: $\frac{2)94}{47}$

4. Do each of these examples both ways: $70 \div 2$; $76 \div 2$; $30 \div 2$; $38 \div 2$; $80 \div 5$; $85 \div 5$.

5. Do these the shorter way only: $45 \div 3$; 72 $\div 6$; $51 \div 3$; $95 \div 5$; $68 \div 4$; $84 \div 6$; $56 \div 2$; $64 \div 4$; $92 \div 4$.

- 1. What is ¹/₈ of 729? We will take 7 bundles of one hundred, 2 bundles of ten, and 9 single sticks.
- 2. Dividing the 7 bundles of a hundred into 3 equal groups or thirds, we find 3)700 that we have 2 bundles in each $^{'}20$ group and 1 hundred over. In this 9 bundle of a hundred there are 10 200 tens; adding these to the 2 tens we 40 find that we have 12 tens. We then 3 divide these into three equal groups, 243 and find that there are 4 tens in each group, and the 9 single sticks divided into thirds gives 3 sticks to each third. adding together the 2 hundreds, 4 tens, and 3 units of each third, we find that 1/8 of 729 is 243. The work we have done with counters is here expressed in figures.
- 3. Here is the shorter way of expressing 3)729 this:
- 4. Work out with counters and express in both forms these examples: $746 \div 3$; $584 \div 4$; $546 \div 2$; $928 \div 4$; $756 \div 3$; $556 \div 2$; $765 \div 3$; $948 \div 4$; $765 \div 5$; $864 \div 6$.

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